Credit-Equity Model Building Blocks

WBS 10th Fixed Income Conference,
Barcelona, 25 September 2014

Vladimir Chorniy, Andrei Greenberg, Lee Moran
Group Risk Management, BNP Paribas

Agenda

- Capturing counterparty risk in the “new world”
  - CCPs, regulators, banks may have different requirements
  - Models: living in the Lego world
- Making the most of available information
  - Market-implied and historical data; structural approach
- Modelling dependence
  - Correlation or cointegration
  - Common drivers (volatility, asset returns,…)
- Joint models of credit and equity underlyings
  - Short-term links: equity-spread correlation
  - Volatility as a common driver: interpretations
  - Structural approach and long-term equity-credit links via ratings
  - Tail dependence and jumps
- Bonus track: two languages to describe fat-tailed equity returns: jump-diffusion and stochastic volatility
- Suggestions, examples and conclusions for “Lego models”

Counterparty risk today

- The way we look at counterparty risk is changing
  - Increased role of clearing and margining (EMIR, Dodd-Frank)
  - Central Counterparties (CCPs)

- Tails become more important, but specific requirements result in zooming in on different areas
  - Still need "classical" counterparty risk calculations: expectation for regulatory capital and CVA, and 90^{th} or 99^{th} percentile exposures
  - With more trades collateralised and cleared, banks focus on higher percentiles over typical slippage / no-control periods for residual risk
  - Long-term stability of the financial system would require extreme events over long horizons to be assessed
  - Cross-asset dependence can become crucial in many of these cases

CCP and Margining

- CCP
  - Client accounts: clearing is segment specific, but close-out is across all segments
  - CCP: rulebook and legal entity specific (e.g., LCH SA vs. LCH Ltd)
  - Cross-asset netting – may be; portfolio effect – definitely
  - Extreme events are expected to be propagating through majority of markets

- Margining (EMIR)
  - Margins (both VM & IM) must be exchanged between counterparties when they are both either Financial Counterparties (FC) or Non-Financial Counterparties above the clearing threshold (NFC+) according to EMIR definitions.
  - Transactions between counterparties where one of them is neither FC nor NFC+ are exempt
  - Legacy – pre-EMIR, but also pre-EBA RTS implementation

- Need to cover existing risk scope and address new elements

Counterparty risk: what needs to be measured and why

Percentile/Horizon → ↓	Short (10d)	Medium	Long (2y+)
Lower (Expectation; 90 - 99%)	Collateralised legacy; NFC-; IM calculation / verification – CCPs/FC, NFC+	Legacy trades and NFC- ("classical"); IM stability – CCPs/FC, NFC+; CCP	Legacy trades and NFC- ("classical")
Higher (above 99%)	Risk above IM covered level CCP/FC, NFC+	Same	Systems stability - stress tests; Regulators; All positions

Lego world of models (I)

- In an ideal world of unlimited resources and no deadlines…
  - "42": develop unified model of everything capturing everything, across assets and risk types
  - Not realistic, and (may be!) not the best approach
    - Shouldn't we understand well each asset class separately before joining them?
    - Shouldn't we understand different types of risk before joining them?
- **Lego approach**: assemble complex model from independent, standalone bits (preferably joining them by pre-designed hooks)
- We are not necessarily saying Lego approach is better, but rather that
  - It is possible and practical, whilst maintaining quality of the final product
    - Project staggered and part resourced
    - Old parts could be recycled
  - It is rich: there is more then one flavour
    - By asset class (equity, credit)
    - By risk type (diffusive/continuous, jump)
    - By risk cause (e.g., credit events, such as migration or default; currency de-pegging, etc.)
  - It is "regulator friendly" - in fact B2.5, B3 and FRTB are biased towards it

Lego world of models (II)

- Traditional Lego approach: by asset class – the classic
  - Different assets treated separately tend to use different models
  - These will require alteration to connect. Examples:
    - Factor models are typical for Credit (due to sparse data issues)
    - Single name (full representation) for Equity
    - Other aspects: WWR (conditionality), default-ability, jumps…
- Why consider alternatives? (risk type and risk cause)
  - Regulatory demands
    - Basel 2.5 introduced IRC: migration and default only modelling – risking by risk cause
      - Risk "type" vs. "cause": if migrations and defaults (cause) is interpreted by risk type, it will cut across: part of the risk is diffusion-type, other parts are jump-type
    - Basel 3: migration, but not default in VaR (to allow 1 year cap in CRR)
    - FRTB: default only IDR (replacing IRC and CRM)
  - Risk Management and Capital Measures
    - Attention to high percentiles only or low percentile/expectation only; short horizons only or long horizons only; forward (CSA) or cumulative (uncollateralised) risk
      - Mixes risk types and risk causes
      - High percentile (extreme risk) for short and long periods can be served by suitable jump process, but for high percentile/long horizon only ,default and, possibly, migrations may suffice
      - Correlated diffusions may be required for large returns, but jump process may cover joint extreme evolution better
    - We will show that it is possible to mix-and-match Lego bricks, even across types (if really needed)

Risk factors and dependence: what to model and how (I)

- Short-term co-movement: returns
  - Returns are best for describing/predicting underlying moves over short horizons
  - Correlated diffusions or common jumps to model joint behaviour
- Long-term predictions: levels (trends)
  - Trends matter much more for long horizons: diffusive moves average out (also $O(\sqrt{\Delta t}) \ll O(\Delta t)$), effect of jumps is short-lived
  - Classical example: long-term mean of an Ornstein-Uhlenbeck process
  - Levels can be used to enforce “pathwise” dependence (e.g., in scenarios with low share prices, spreads should be high)
    - In equity long-term trend and low/high levels are hard to separate

- We look at equity-credit models in cross-asset context
  - Relevant for equity financing, repo, stock lending/borrowing
  - Some well-known fundamental relationships (jump-to-ruin, low share prices – wide spreads, etc.)
  - Structural link

Risk factors and dependence: what to model and how (II)

- Modelling quantities
  - Equity returns, share price levels
  - CDS spreads preferable to hazard rates in risk context due to observability
    - Hazard rates generate “price-able” scenarios
  - Equity volatility
  - Asset returns

- Dependence
  - Correlation (and/or common jumps) of stochastic drivers for returns
  - Cointegration, or mean-reverting “spread” between levels
  - Regime shift: time- or state-dependent correlation (e.g., higher for extreme returns than in the middle of the distribution)
  - Common drivers: if correctly incorporated, leads to better models
    - Fundamental causality: changes in the same external quantity driving changes in equity and credit
    - Mathematical stability: if a common driver exists, modelling it + relationships will produce a more robust model
    - Example: Merton’s model, where equity = call, debt = put (+cash) on asset value

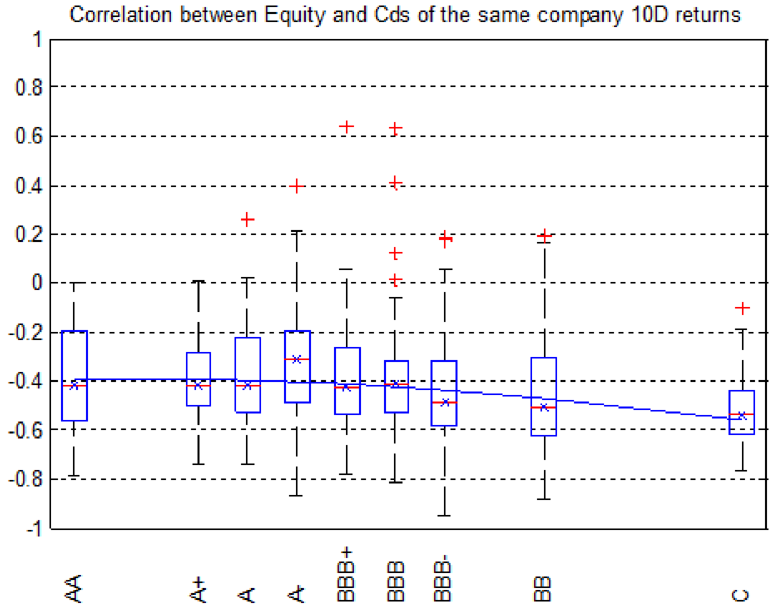
Market information

Type	Historical	Market-implied
Equity	Equity prices and returns, volatility of returns, jumps in returns	Volatility (ATM, OTM/smile) [Equity]
Credit	CDS spreads, returns and jumps; ratings	CDS spreads
Asset returns	☑	☑

- Risk models operate in the “real-world measure”, so historical data are generally preferred for calibration
- However market-implied information has the advantage of a forward view
  - Instantaneous CDS spreads contain information about default probabilities and can be used to predict sudden moves in equity
  - Implied volatility represents market view of future volatility of returns
- Asset (firm value) returns: strictly speaking not market data, but can be useful
  - Potential common driver for equity and credit (via structural models)
  - May be used to model rating transitions

Short-term equity-spread return correlations (I)

- Negative correlation expected market-wide: tightening of spreads is associated with increased equity returns as share prices go up
- For individual names, some dependence on credit quality may transpire
  - Median correlation between equity and spread returns of the same name is -41%
  - Poorly rated names show stronger link:

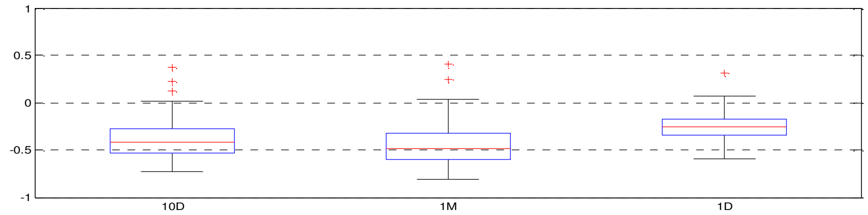


  - How stable is the pattern? (variation margined or CSA-based risk)
  - Is it “Lego-compatible”?

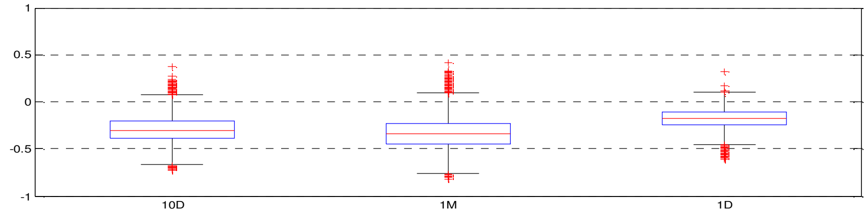
Short-term equity-spread return correlations (II)

- Returns:
  - 1D vs 10D vs 1M

Correlation: Same name 10 Days – 1 Months – 1 Day returns



Correlation: all names 10 Days – 1 Months – 1 Day returns



- Is it stable?
  - Yes: see 2009-13 vs. 2007-11

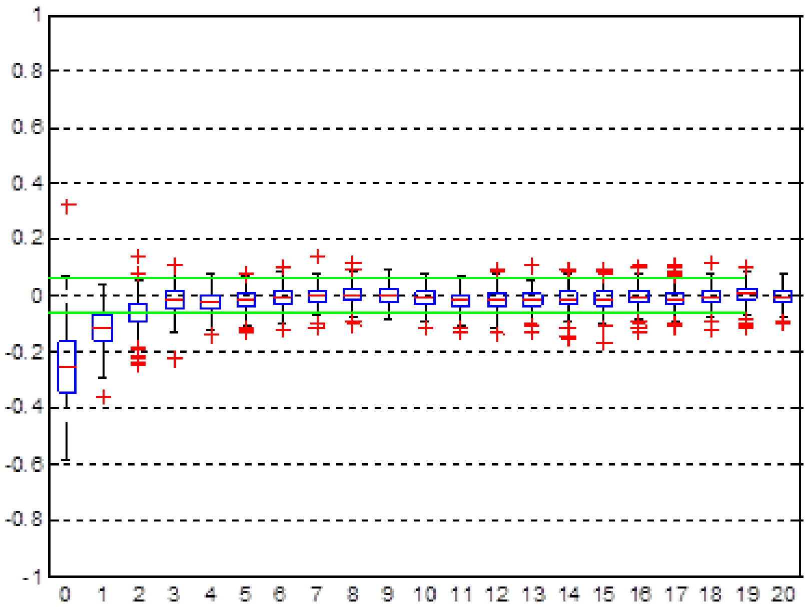
	AA	A+	A	A-	BBB+	BBB	BBB-	BB	C
2009/04-2013/03	-15%	-15%	-13%	-11%	-18%	-16%	-18%	-27%	-30%
2007/04-2011/03	-16%	-12%	-11%	-14%	-16%	-19%	-15%	-25%	-22%

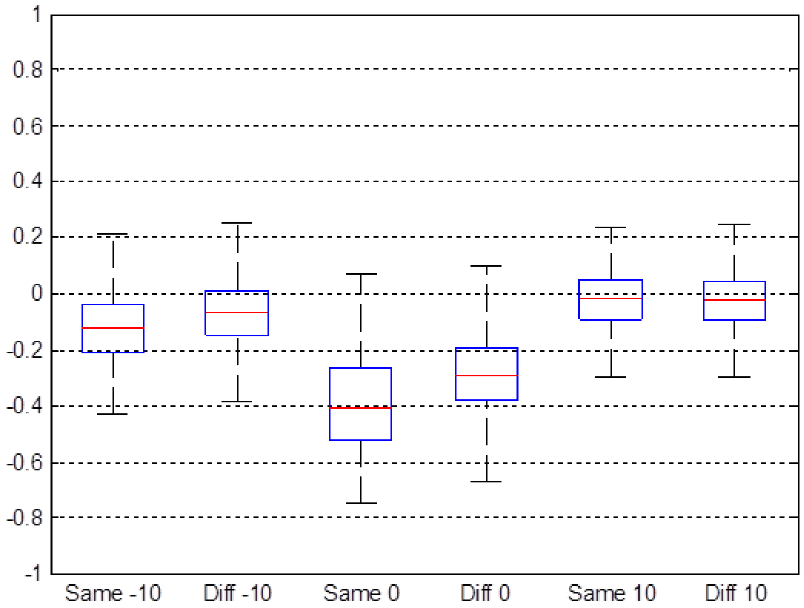
Median of correlations between CDS and Equity residual returns

- Is it Lego compatible for factor vs. single name?
  - Yes, for example:
    - introduce artificial equity "systematic" time series and "equity residuals" in credit
    - in factor model for credit, break idiosyncratic factor in two parts: "pure" idiosyncratic part and issuer specific component common with equity

Short-term equity-spread returns: why 10 days?

- In counterparty risk models, 10 days is the risk period common to most CSAs
- Market evidence points towards equity returns *leading* CDS returns



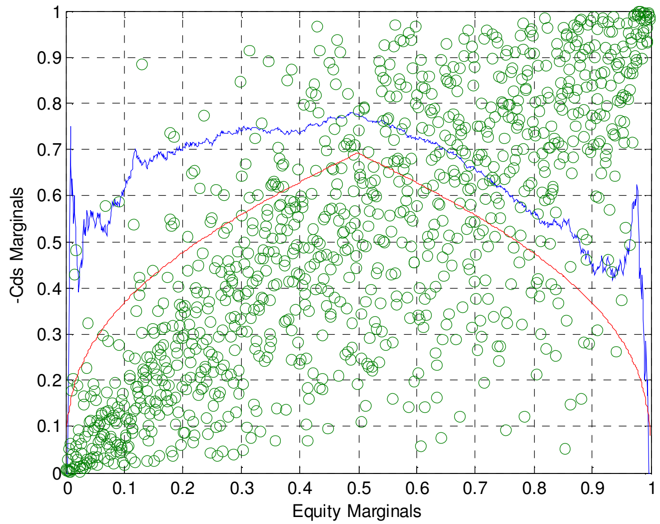


- Lead-lag effects disappear by 10 days, as cross-correlation plots show

Lego: next steps

- What additional “bricks” to consider?
  - Dependence between extreme moves
    - Jumps
    - Non-Gaussian copulae
    - Overlay of driver/process choice (e.g. joint default process; factor model)
    - Depends on extreme moves per underlying
  - Extreme moves per underlying
    - How extreme and over which time horizon?

Single market factor copula illustration – green circles are a joint equity and CDS [negative] returns of systematic component, plotted in the two marginal cumulative distributions. The density of observations is higher around the x=y line showing that they are generally correlated. The blue and red lines are *tail dependence coefficients* of empirical and Gaussian copulas respectively, indicating that extreme returns are empirically more “correlated” than Gaussian copula predicts.



- Our choice will be better informed if we understand “why”. We need to know the drivers. What are the choices?

Equity-credit dependence via a common driver

- Classical example: structural, or firm value, models
  - Equity = long call on assets of the firm struck at the face value of debt
  - Bond = short put on asset of the firm (same strike) + cash
  - Given a model for the asset value/return process and its volatility, we get the behaviour of both equity and credit
  - Various flavours developed over the years
    - Original Merton (1974): GBM asset returns, single-period model
    - First-passage models – Black-Cox (1976): time dependent, with default barrier
    - Adding jumps to the asset return process : Lipton (2001)
    - Stochastic default barriers: e.g., Leland-Toft (1996), Brigo-Tarenghi (2005)
    - Two-factor capital structure model: Hurd & Zhou (2011)
- Drawbacks of firm value models
  - Asset returns are not observable: hard to calibrate and test assumptions
  - Capital structure dependence is too rigid: credit and equity are options on the same underlying, which restricts the dependence
  - Short-term spread problem: credit spreads go to zero as $T \to 0$ in purely diffusive models, so jumps and/or complicated barriers are necessary
- Alternative common driver models?

Can volatility be a common driver for equity and credit?

- Three interpretations of dependence between CDS spreads and equities, and volatilities (i.e., volatility as the driver)
  - “Pure Merton”: link via asset (i.e., firm value) volatility, which is a measure of investment risk. Equity vol is function of asset vol or is asset “vol of vol”.
  - “Leverage effect”: general term for equity volatility inversely related to equity returns
    - Volatility is measure of [investment] risk: higher risk requires higher return, so lower stock price
    - CDS also reflects the riskiness of investment
    - Volatility is in effect “priced“ (sometimes counts as another interpretation)
  - Equity volatility embeds a measure of probability of jump-to-default
    - Excess return of equity over risk-free rate compensates for possible default
    - CDS reflect market view of the default probability.
- Other interpretation of volatility: market indicator
  - If strong enough, it is equally good as state variable
  - Indicator of investment risk: again a flavour of leverage, but reversing the causality
    - Financial leverage: debt-to-equity ratio of the firm, so drop in stock price increases leverage ratio
    - Higher leverage $\Rightarrow$ riskier stock $\Rightarrow$ more volatile stock
  - Reaction to change:
    - While market is looking for a new equilibrium (should also show link to liquidity and trading volume), it displays higher uncertainty, hence volatility increases

Volatility as a common driver for equity and credit (I)

- What does each interpretation predict in terms of form of dependence?
- We stated that it would be good to separate short term behaviour (returns) for long term (levels or trends). Here are some caveats:
  - CDS level as absolute number (high or low) has a meaning
  - Similarly for volatility
  - Stock price can only be "high" relative to something:
    - "Buoyant" market means *rising* stock prices, which is not high level but positive trend, or persistent positive return
    - But once "high" levels are reached, if market stays there, then we have high levels with ~zero average returns
    - Thus returns, trends and levels for stock are hard to separate

- If exact functional relationship exists between two variables (e.g., Merton), it will show perfect link both for returns and levels, both short- and long-term
- **Partial** validity may assert itself *on shorter or longer scale only*

- What are the predictions? (Ideally should be different for different interpretations)

Volatility as a common driver for equity and credit (II)

- Merton and firm value volatility:
  - "Time value" of Merton options: changes convexity of equity-credit "hockey stick"
    - $\Rightarrow$ Volatility increase means both equity and CDS spread increases
  - "Vol of asset vol": stochastic volatility model predictions
    - $\Rightarrow$ Fatter tails make options more expensive, so both equity and CDS spreads increase (but less)
    - Equity investors like the risk (long vol), bond investors do not (short vol)
  - So for Merton, **Vol *up*** $\Rightarrow$ **CDS *up*, Equity *up***
  - "Time value" effect for the vol link is weaker than Equity-Credit impacts predicted by structural model, so any impact is expected to be visible over *longer term*
- Leverage: more immediate dependence, expected to dominate short term
  - Expect CDS, equity and volatility *returns* to be correlated
  - In "hockey-stick" interpretation, where "Merton" reduces convexity of the graph, "Leverage" moves to another curve
  - So for leverage, **Vol *up*** $\Rightarrow$ **CDS *up*, Equity *down***
- Link via default information:
  - CDS, equity and volatility *levels* should be related
  - **Vol *up*** $\Rightarrow$ **CDS *up*, Equity *down***
- Change indicator: any large return (both signs!) leads to increase of volatility
  - Either *return-level* or *return-return* link, only short term
  - **CDS *up or down*, Equity *up or down*** $\Rightarrow$ **Vol *up***

Volatility as a common driver: some evidence in literature

- Bednarek (2006): empirical test for an extension of Merton model
  - Time-dependent asset volatility produces more realistic (i.e., higher) credit spreads
- Campbell & Taksler (2003): booming stock market in 1990s accompanied by rising corporate bond yields – counterintuitive?
  - Optimism of equity investors not shared by bond investors
  - Volatility may be the key: more upside for shareholders, more risk for bondholders
- Martin (2009): equity-credit trading strategies
  - “Hockey stick” dependence between equity returns and spreads, moves up and down with change of equity volatility
  - Risk management and trading implications
- Cremers et al. (2008) : implied volatility affects credit spreads
  - Both ATM and OTM/skew explain a significant part of CDS spread levels
- Carr & Wu (2009, 2011): economic similarity between CDS and deep OTM equity puts
  - Replicate unit recovery contract from CDS and American puts
  - Joint framework for estimating CDS returns and implied equity volatilities

Volatility vs. equity/credit: relationships we can measure

- Use linear regression at first: R^2 to indicate strength of relationship
- According to our paradigm, need to analyse various combinations
  - CDS and equity
  - Levels and returns
  - Implied and historical volatility
  - ATM volatility and skew/OTM volatility
- Questions
  - Are CDS levels stationary?
    - Cremers et al. (2008) argue to the affirmative
  - What to use for OTM implied volatility?
    - “ATM skew” vs. “DOOM put vol”
  - If standard deviation of historical returns “predicts” ATM implied volatility, then what historical data have information about OTM volatility or skew?
    - Stochastic volatility models: correlation between equity returns and their variance (Heston)
    - Jump-diffusion models: average size and intensity of jumps in equity returns (historical estimates less stable)
- “Equity levels” ?? (more on this later)

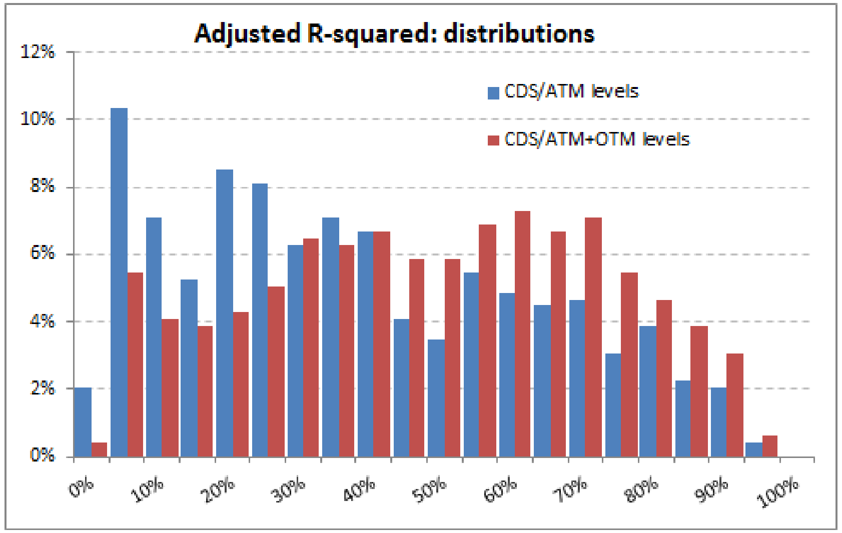
Measurement: the boring details

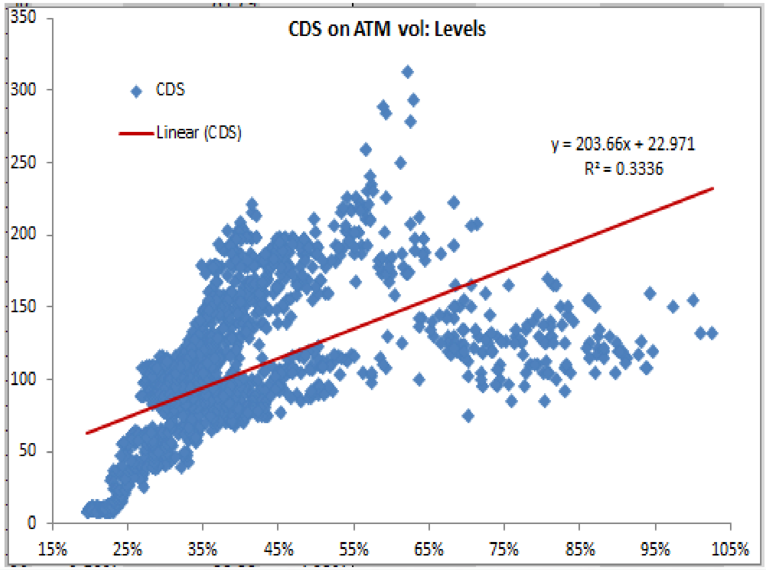
- Universe:
  - Ca. 500 names from major international equity indices with liquid CDS
  - Time series from September 2006 to August 2013
  - More liquid names subset: 160 names

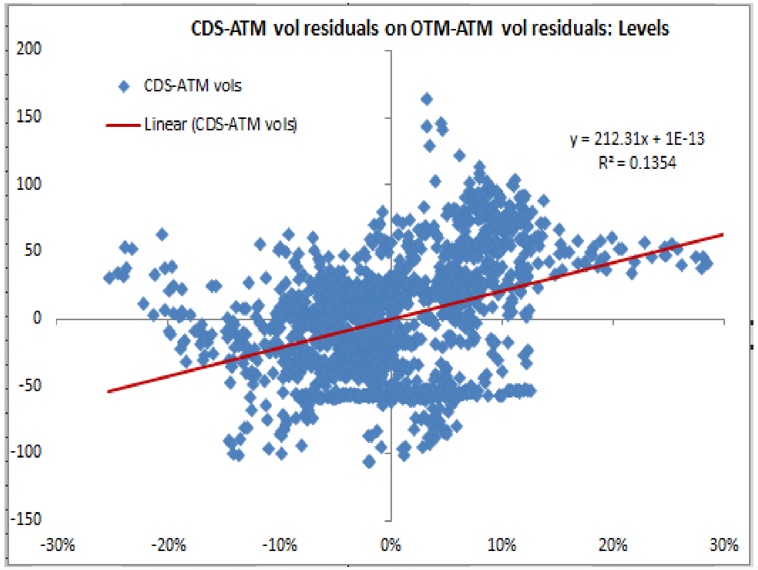
- Implied volatilities:
  - 6m option implied ATM vols
  - Deep OTM put vols (extrapolated to 10% strike)
  - Skew as (ATM – OTM) / (100% - 10%) < 0 for equity
- Historical volatilities:
  - Standard deviation of 10-day returns, estimated over 6 months and annualised
  - Correlation with variance measured over 6m window
  - Time-averaged jump measures over 6m windows
  - To distinguish effects of OTM volatility (and proxies), regress residuals:
    - E.g., residual from regression of CDS spread on ATM vol projected on residual from regression of OTM on ATM vol gives an estimate of the effect of OTM vol on CDS spread
    - Visually more informative than using adjusted R^2's

CDS spread and volatility (I): levels on levels - implied

- Median R^2 is 32% for ATM vols, going up to 46% when OTM is added
- 45% and 55%, respectively, for the subset of more liquid names
- Distribution of R^2 clearly shifts to the right when skew is added
- Regressions shown for Deutsche Bank: positive slope means: high vol → wide spreads

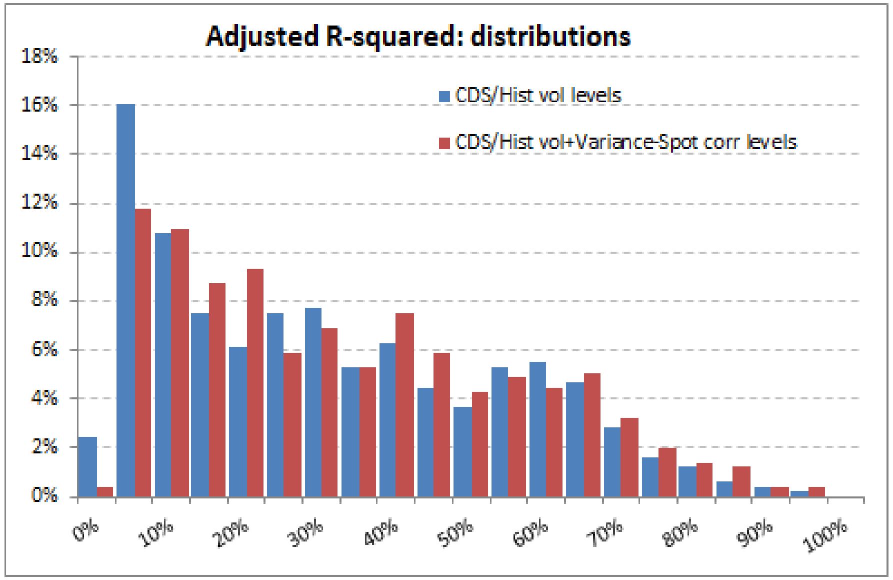


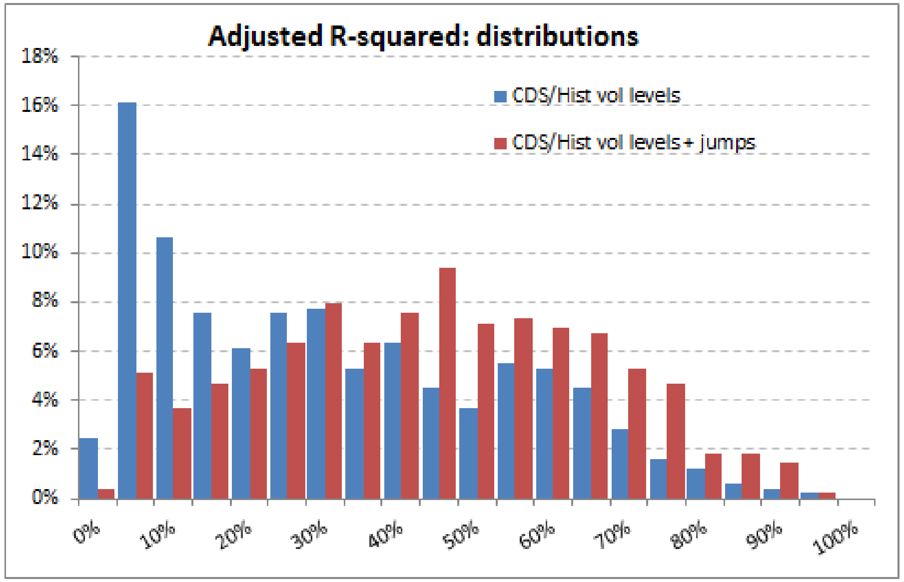




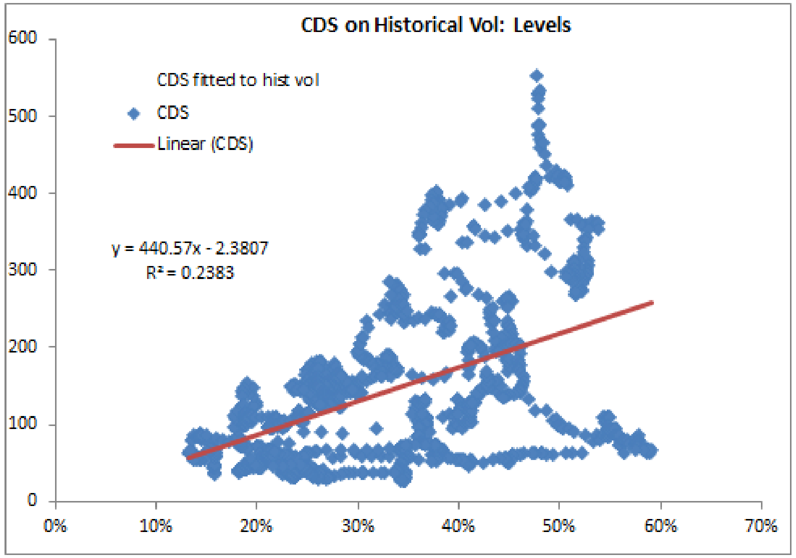
CDS spread and volatility (II): levels on levels - historical

- Weaker dependence on historical vol: median R^2 is 24% (34% for subset of more liquid names)
- Jumps explain residuals better than correlation of returns with their variance
  - Median R^2 goes up to 41% (47% for liquid) with jumps, vs. 27% (41% for liquid) with variance-to-returns correlation
  - Jump risk embedded in CDS or non-stationarity of average jump size time series?
- Related question: which language is better at describing equity dynamics, jump-diffusion or stochastic volatility? (separate study)

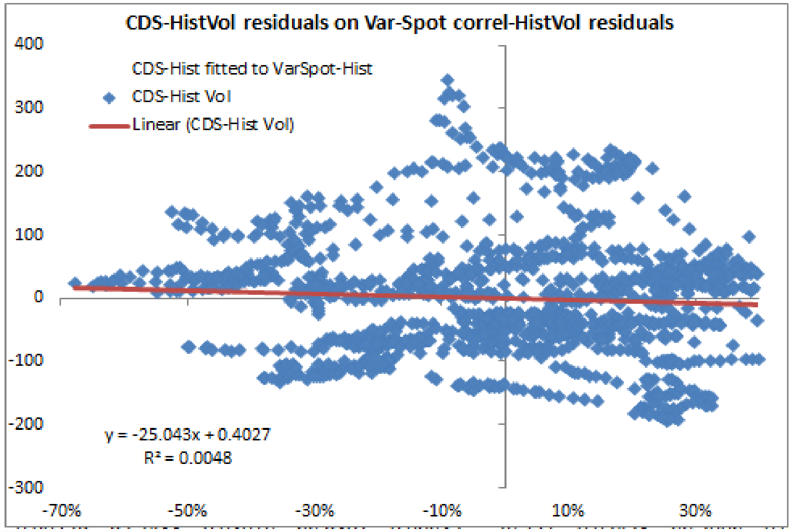


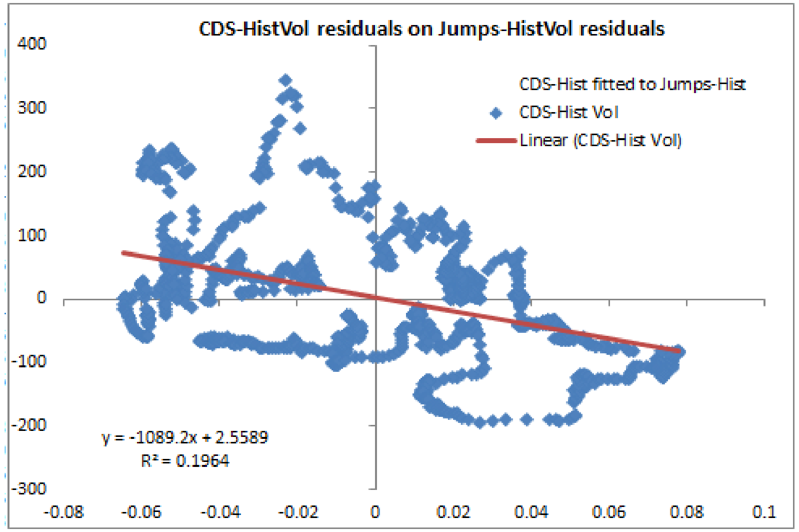


CDS spread levels on historical volatility levels: example 1

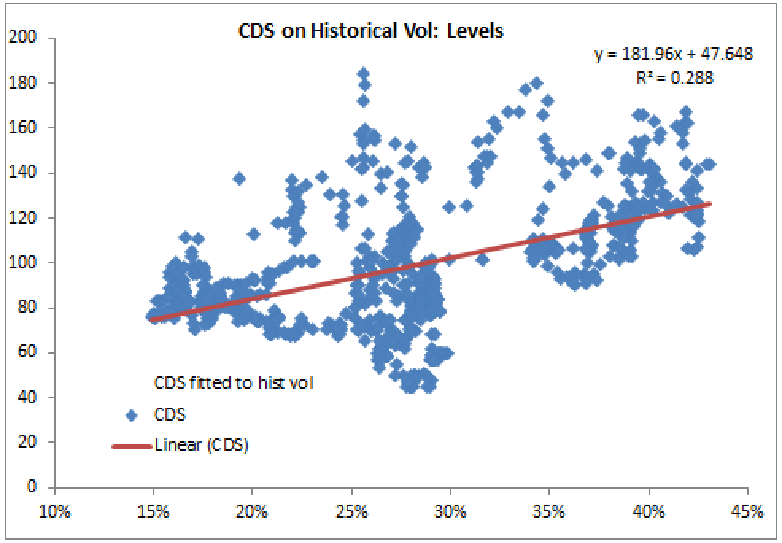


Computer Sciences Corporation: averaged jumps explain CDS residuals better than variance-to-returns correlation

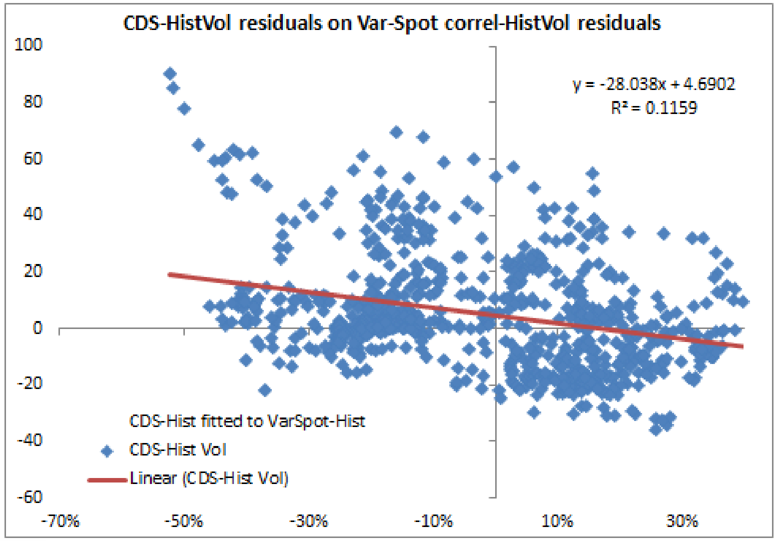


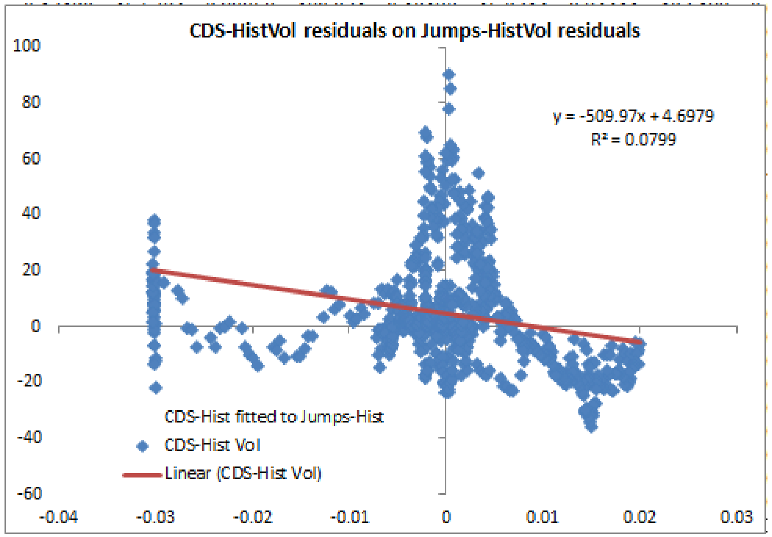


CDS spread levels on historical volatility levels: example 2



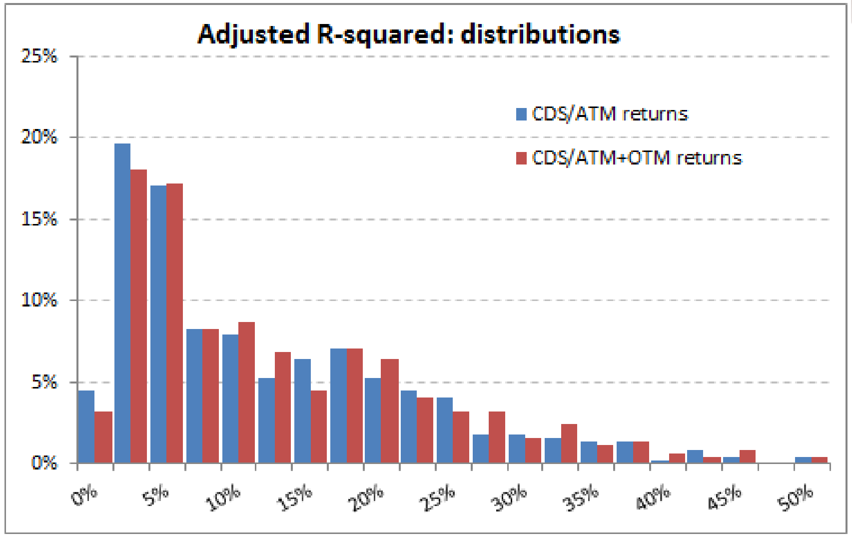
JP Morgan Chase: variance-to-returns correlation explains CDS residuals better than averaged jumps

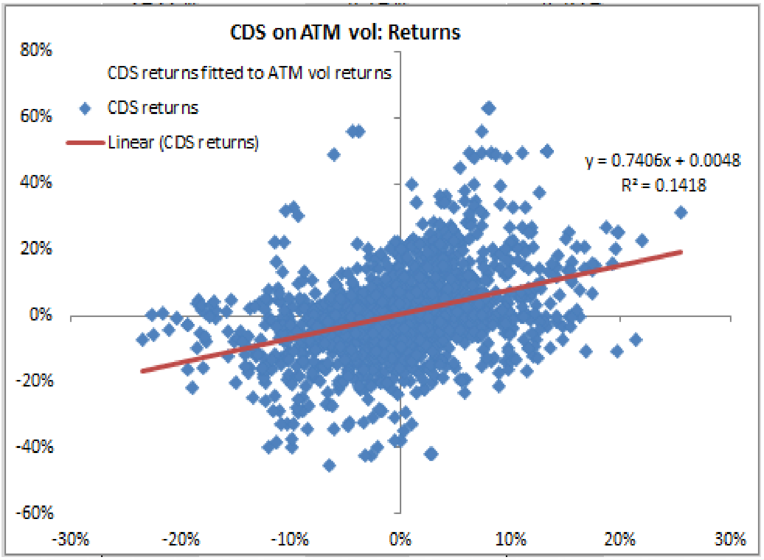


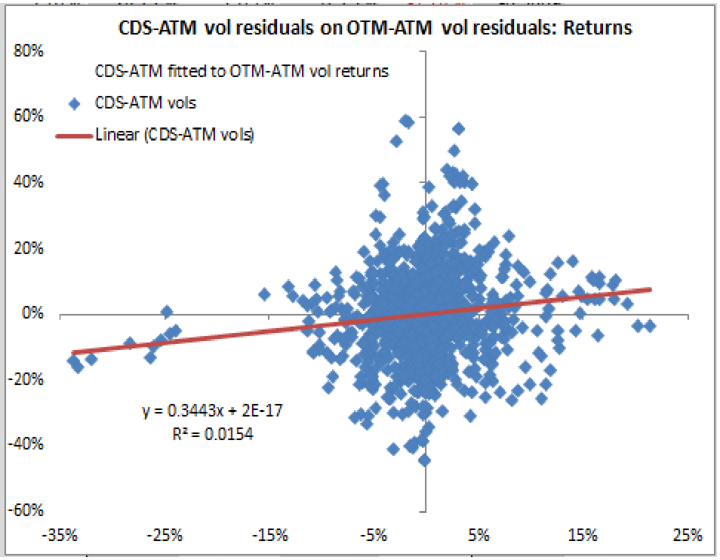


CDS spread and volatility (III): returns on returns - implied

- Very weak dependence: median R^2 is <8% for ATM vols, OTM adds nothing
- Slightly higher median R^2 (20%) for the more liquid names – still visibly smaller than levels, and no OTM effects
- Less clustering in the data than for levels
- Regressions shown below for Next plc: positive slope, just as for levels

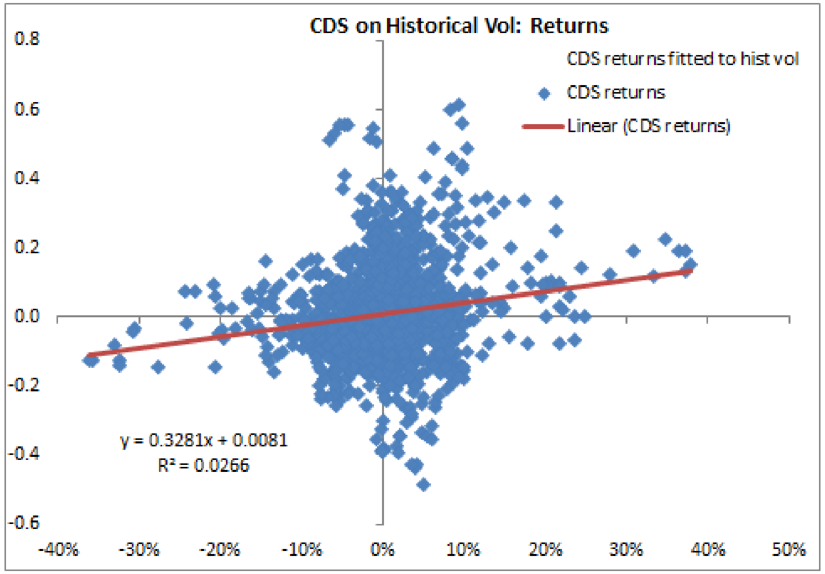


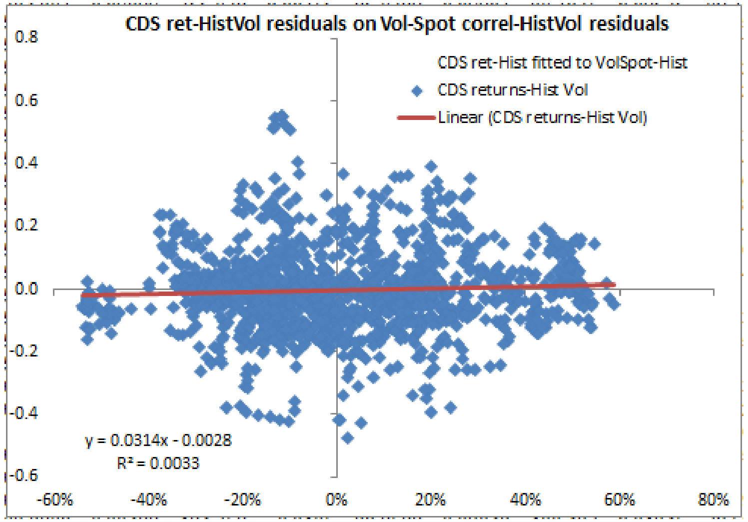


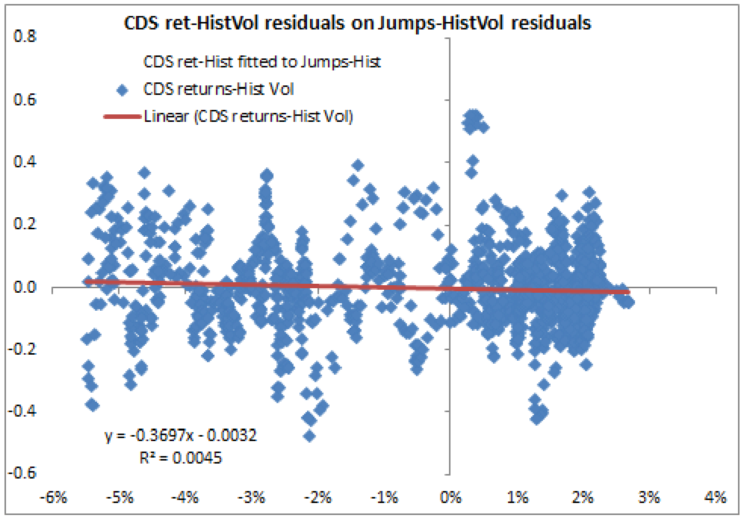


CDS spread and volatility (IV): returns on returns - historical

- Hardly any dependence at all: median R^2 is 1-3%, no matter how many historical vol returns are taken or which skew proxy is chosen
- Picture does not change for liquid names
- Regressions shown for Accor SA

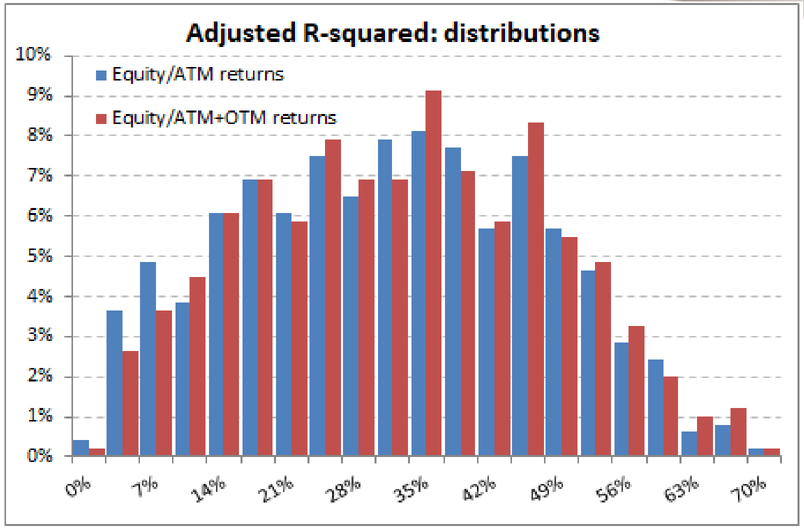


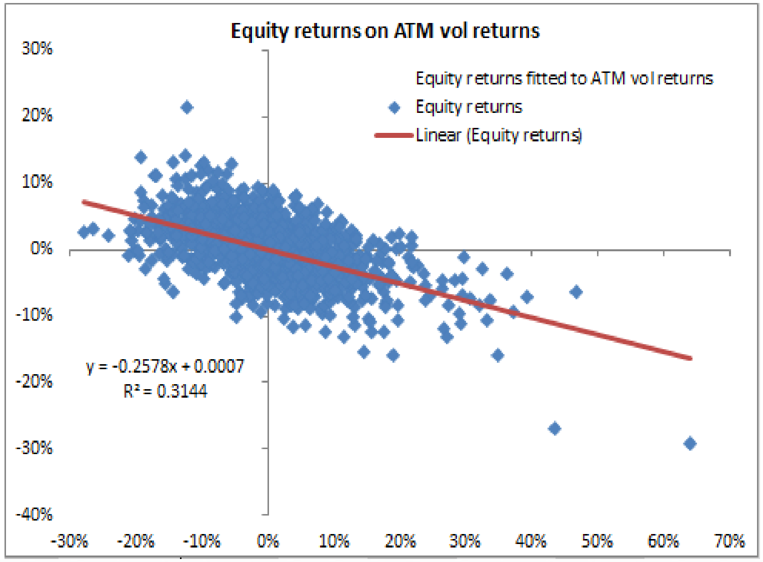


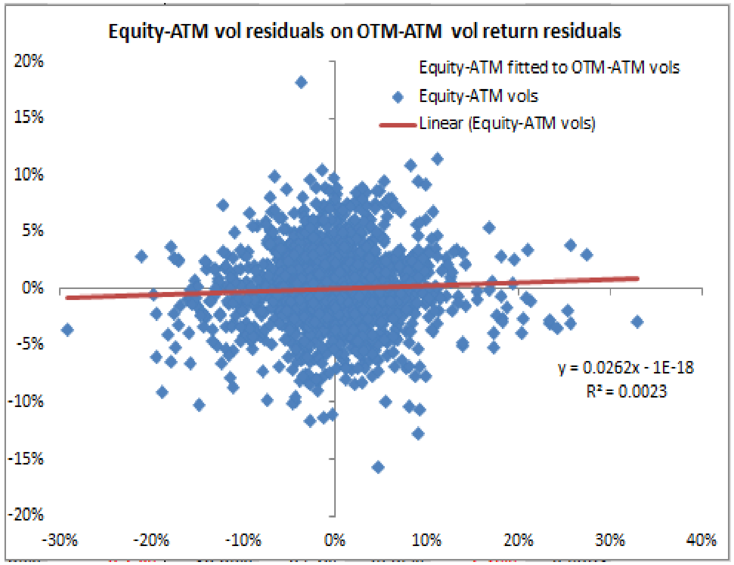


Equity and volatility (I): returns on returns - implied

- Median R^2 is 30% for ATM vol returns, OTM adds nothing
- Slightly higher median R^2 (40%) for the more liquid names, still no OTM effects
- Seems like implied volatility skew plays no role in equity returns, only ATM does
- Regressions shown below for AT&T : negative slope means: high vol returns → negative equity returns

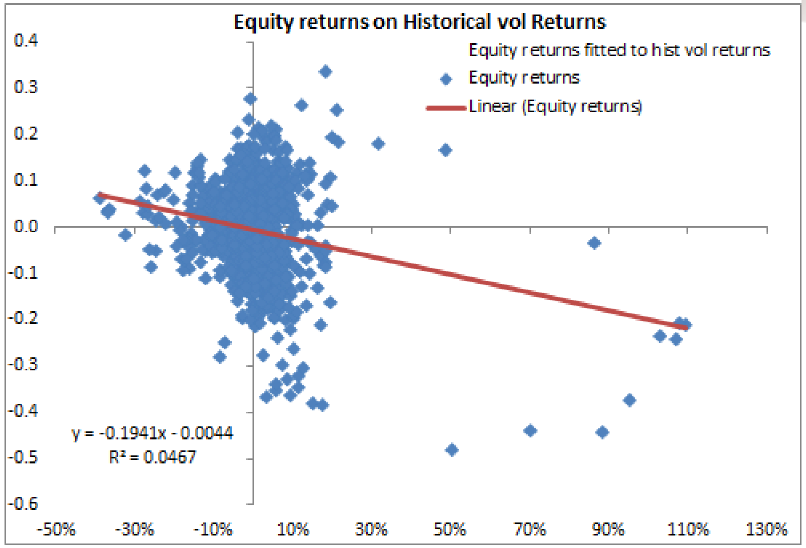


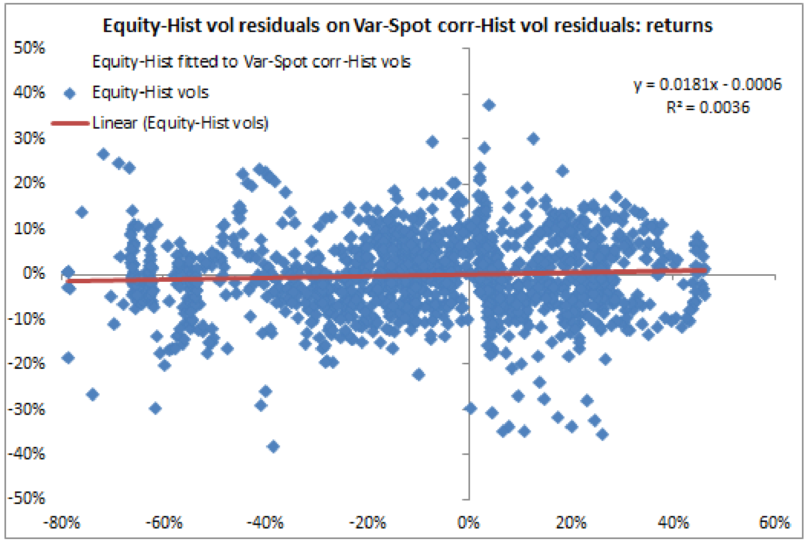


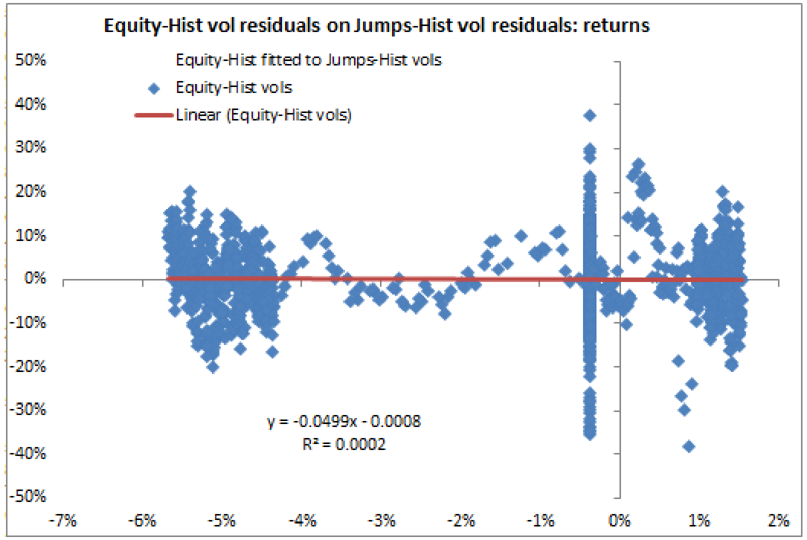


Equity and volatility (II): returns on returns - historical

- Hardly any dependence: median R^2 is 3% for historical vol returns, increasing to 4% with either one of the two historical skew proxies
- Very similar numbers for more liquid names
- Same situation as for CDS returns
- Regressions shown for Toshiba Corporation





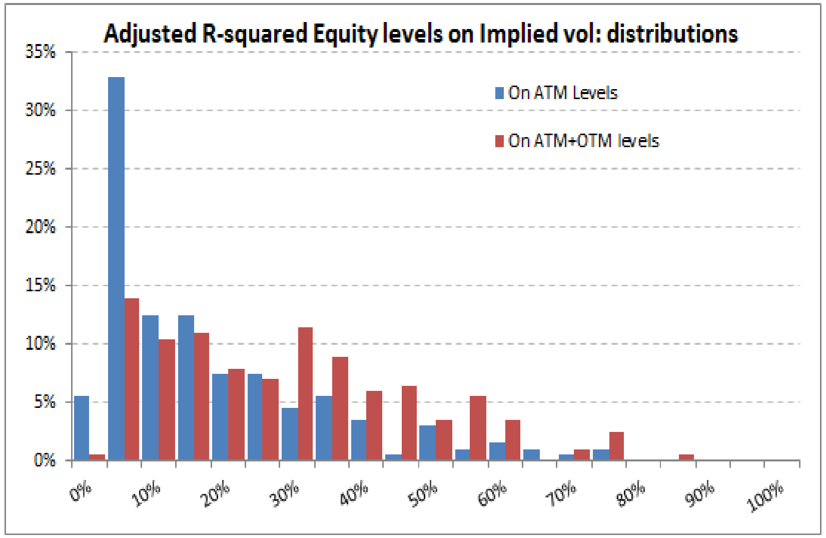


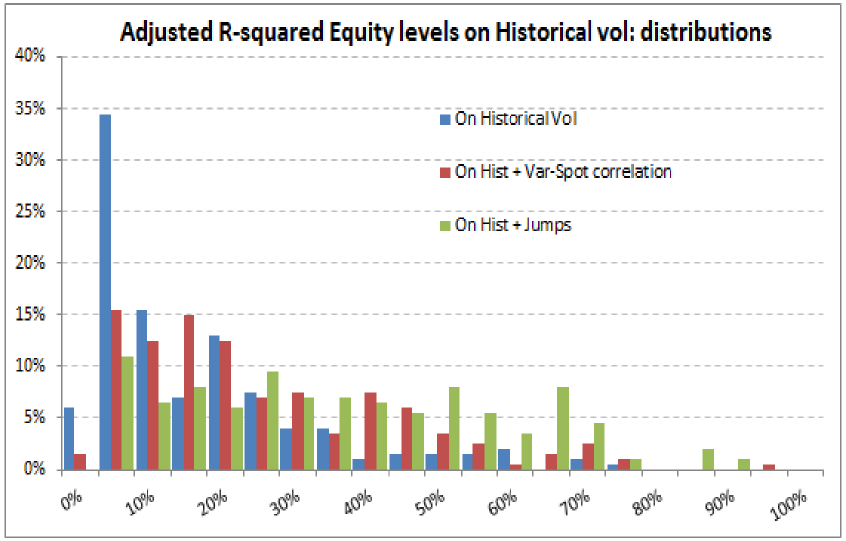
Missing: equity levels on volatility levels?

- CDS spread analysis demonstrates that levels regress much better than returns, especially when historical vols are used
- Problem: equity prices are meaningless for OLS regressions!
  - Dispersion across markets and currencies, no uniform base for comparison
  - Equity price time series are non-stationary, so spurious regression likely
- Can we take a stab at designing a synthetic "equity level"?
- Step back: why are equity prices not meaningful?
  - Share price is not a good indicator of a company's "investor value": doubling the firm's assets and liabilities will increase share price, but not reduce its riskiness
  - CDS spreads (price of default risk) don't have this "size effect", nor do equity returns
- Idea: come up with an appropriately *normalised share price*, to make the measure comparable across different types and sizes of companies
- Proposal: divide share price by the price of the index it belongs to
  - Better statistical properties of the time series expected
  - Market cap weighting helps: normalisation brings companies to more equal footing
- Statistical tests show improvement in stationarity, although not for all names and indices (could be due to index weighting rules?)

“Equity levels” on volatility levels regressions

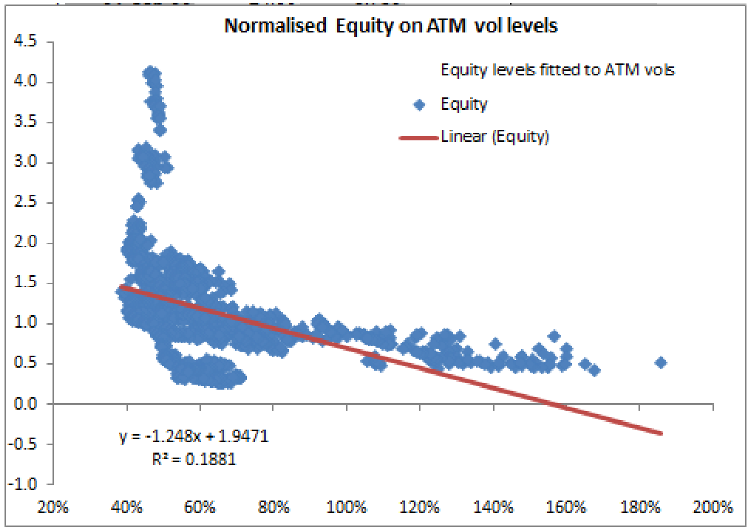
- Performed on a subset of ca. 200 names with near-stationary “equity levels”
- See improved R^2 over historical vol returns, but not over implied vol returns:
  - Median R^2 is 10% for implied ATM vols and 9% for historical, increasing to 24% with OTM vol and to 17% with variance-to-returns correlation and 31% with jumps
  - Compare with 30% median R^2 for equity returns on ATM vol returns, 3-4% R^2 on historical vol returns, and no effect of skew or its proxies
  - Still smaller than the “levels” regression for CDS spreads
- Note: slope can have any sign – support different interpretations

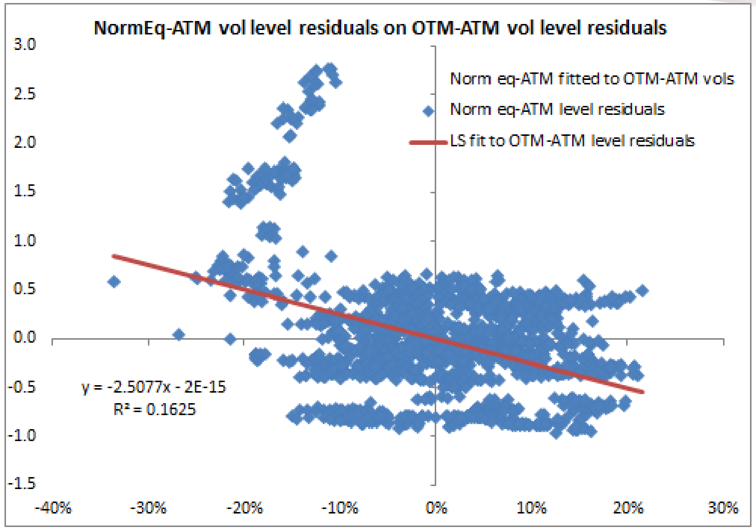




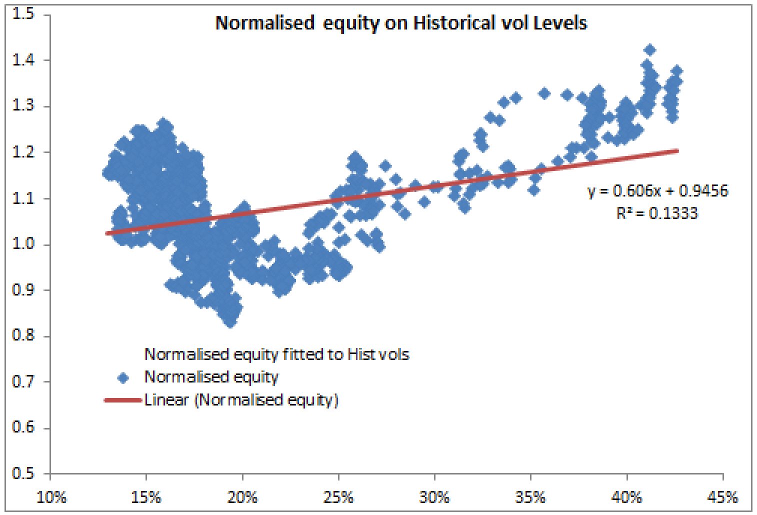
Equity levels on volatility levels: examples

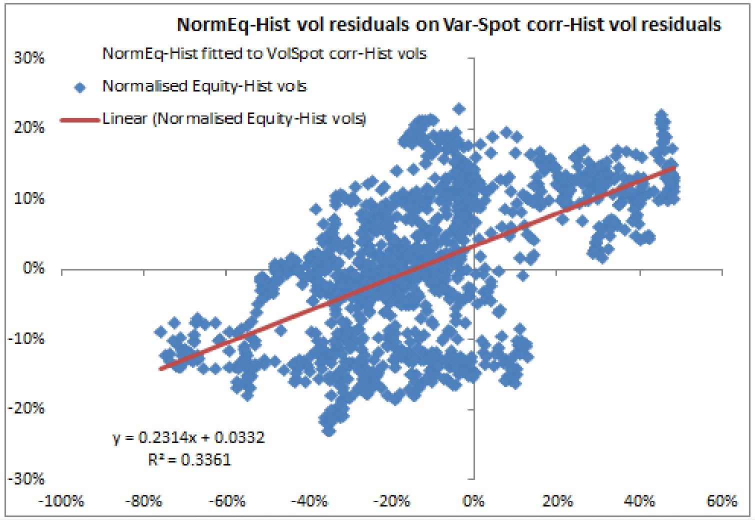
Implied: AMD





Historical: Verizon





CDS and equity regressions on volatility: is it working?

Regression R^2		Implied vol		Historical vol		
		ATM	ATM+OTM	Hist	Hist+Correl	Hist+Jumps
Equity	Returns	30%	31%	3%	3.5%	3.5%
	Levels	10%	25%	9%	17%	31%
CDS	Returns	8% ✸	8% ✸	1.5%	2.5%	3%
	Levels	32%	47%	24%	27%	41%

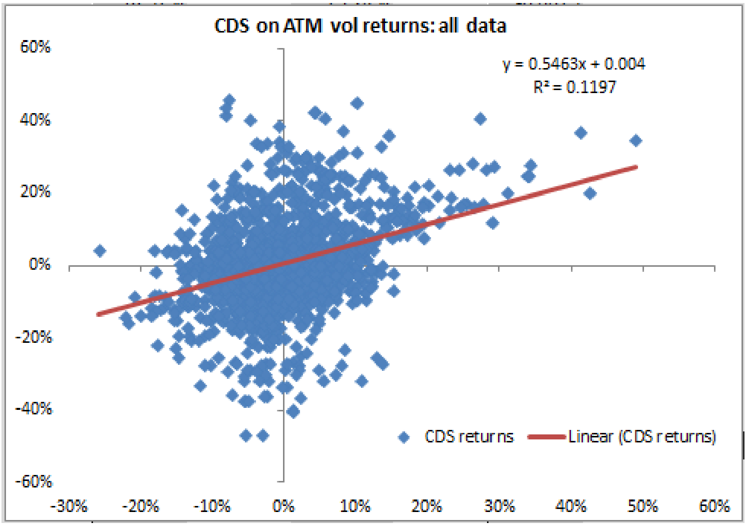
- Some evidence of dependence between volatility and both CDS spreads and equities
  - Stronger for implied than for historical, except for jump measures in levels
  - Volatility skew measures are important for levels, but not for returns
  - Better for CDS levels than for CDS returns; but conversely for equity
  - Jumps as indicator of historical information for volatility skew play a part in explaining levels for both CDS and equity
  - Liquid subset emphasizes the same pattern (one exception: ✸ is up to 20%)

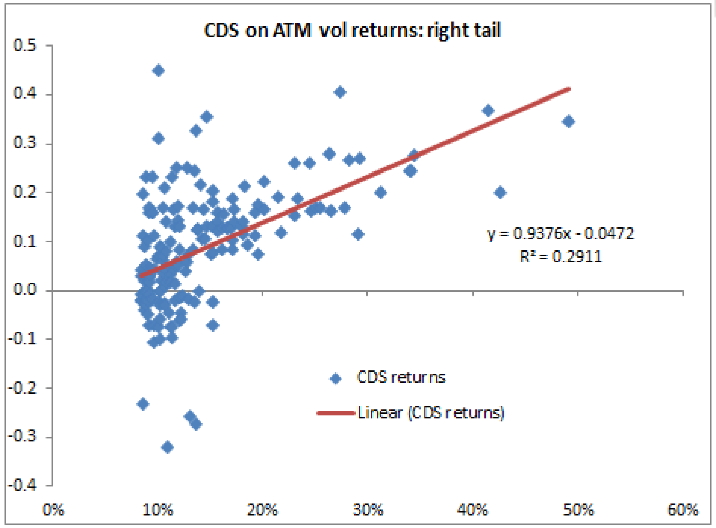
Looking further: tail regressions

- Dependence of extreme returns can differ from "normal" returns
  - For example, even low-correlated names can start dropping together in a crisis
- We perform regressions on the top and bottom 10% returns only, and compare with main results ("central" return scenarios)
  - One tail at a time, to avoid artificial "R^2 inflation"
  - Use CDS-on-vol returns as an example (weakest dependence)
  - Look for a pattern such as the one shown on the next slide
- Some evidence of different dependence strength observed
  - More of high R^2's in the "right tail": stronger dependence between high positive vol returns and high CDS spread returns, especially for historical vols
  - More of low R^2's in the "left tail": weaker dependence between high negative returns
  - Consistent with the "crisis" intuition, but median R^2 still only goes up to ~10%
    - CDS on implied vol returns: from 7.7% for all returns to 10.3% for high returns
    - CDS on historical vol returns: from 2.5% for all returns to 10.5% for high returns
- Fact: linear models are not very good at capturing tail dependence…
  - Examples do not show volatility reaction to "any change" (similar for jumps study)

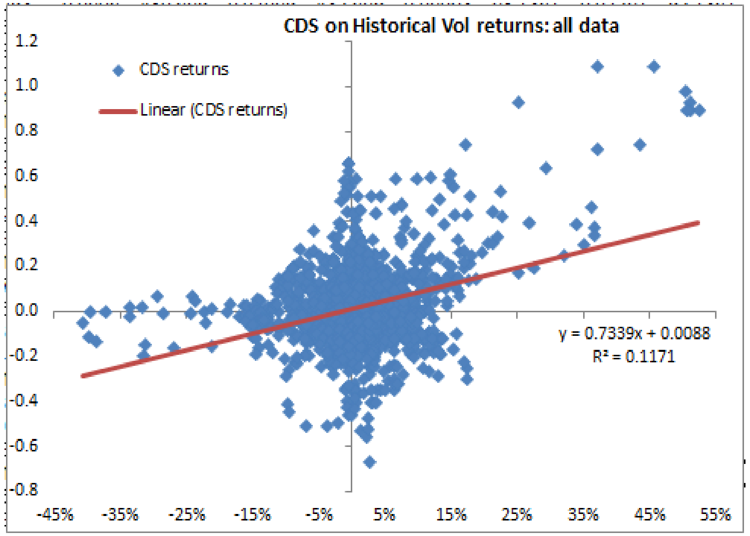
CDS spread returns on volatility return tails: examples

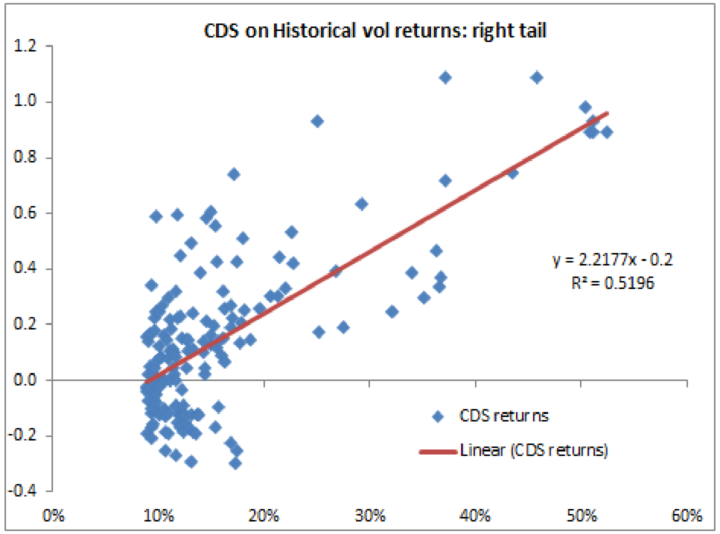
Implied:
BAT





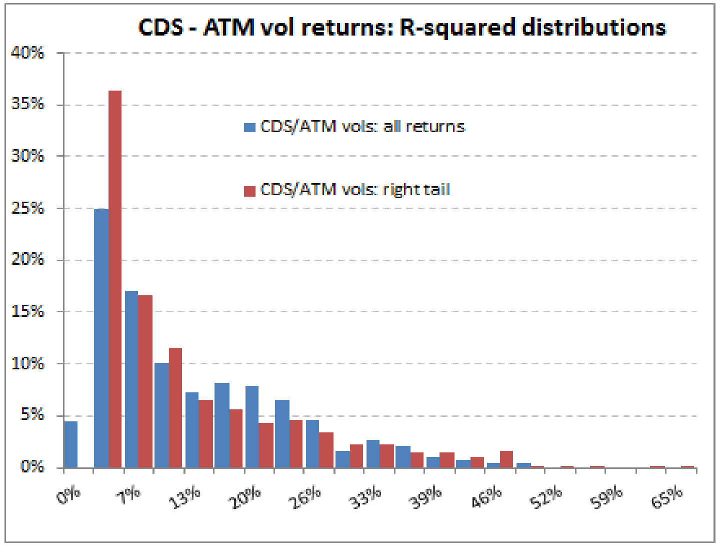
Historical:
Heidelberg
Cement

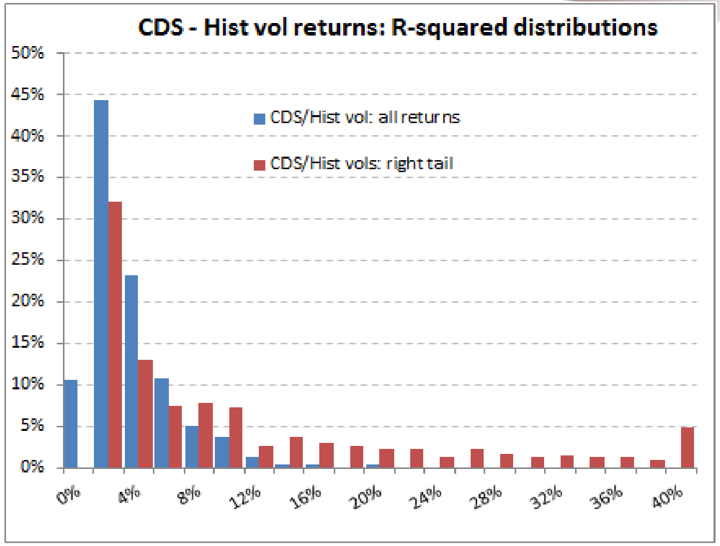




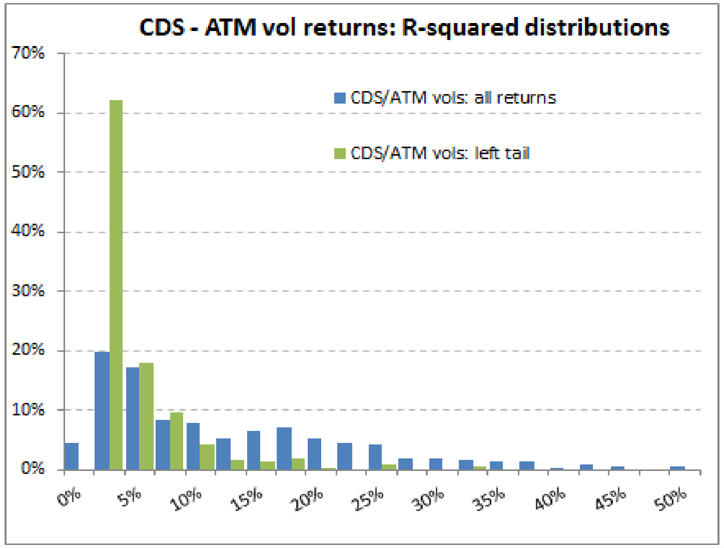
CDS spread returns on volatility return tails: R^2 distributions

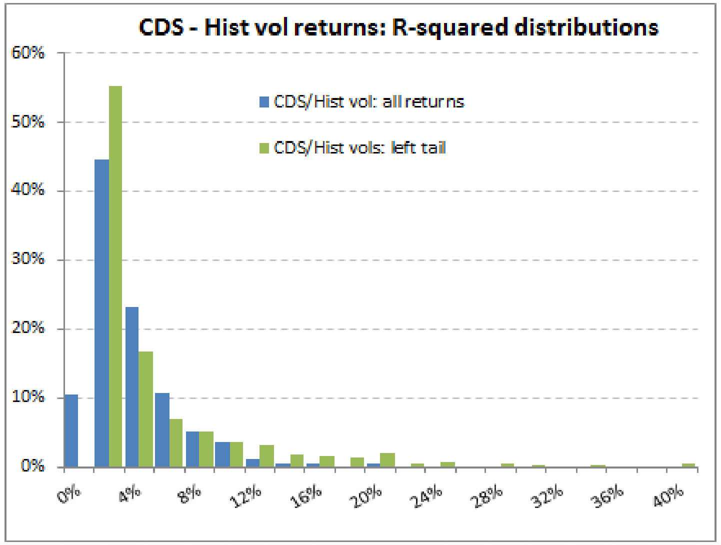
Right tail
(high returns)





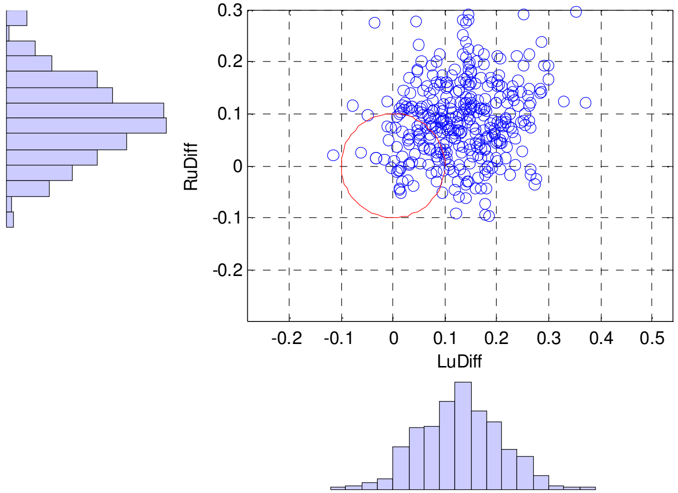
Left tail
(low returns)

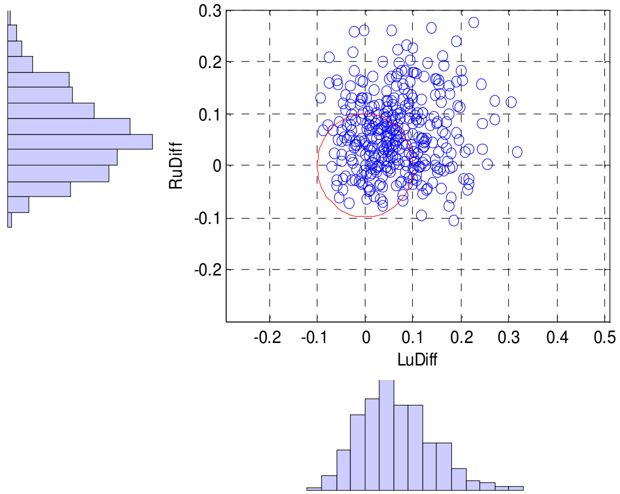




Tail dependence: beyond regression

- Measure empirical 10% left and right tail dependence between CDS and equity returns vs. corresponding Gaussian copula-implied numbers
- Scatter-plot the differences, against the 95% confidence interval around purely-Gaussian tail dependence (i.e., around zero difference)
  - Total returns on the left plot, residual (idiosyncratic) returns on the right plot
- Data suggest both systematic and idiosyncratic dependence of extreme returns which is different from Gaussian
  - Can add common systemic jumps + company-specific jumps in equity and CDS



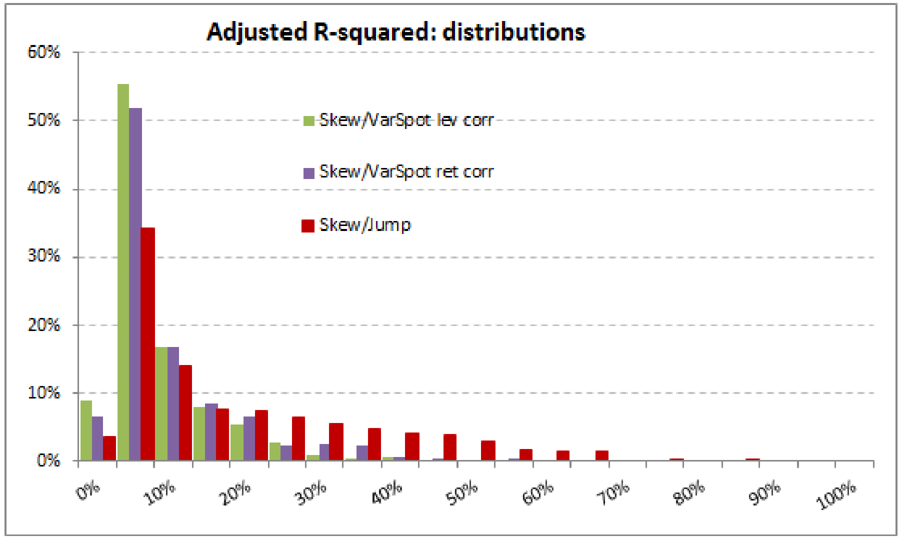


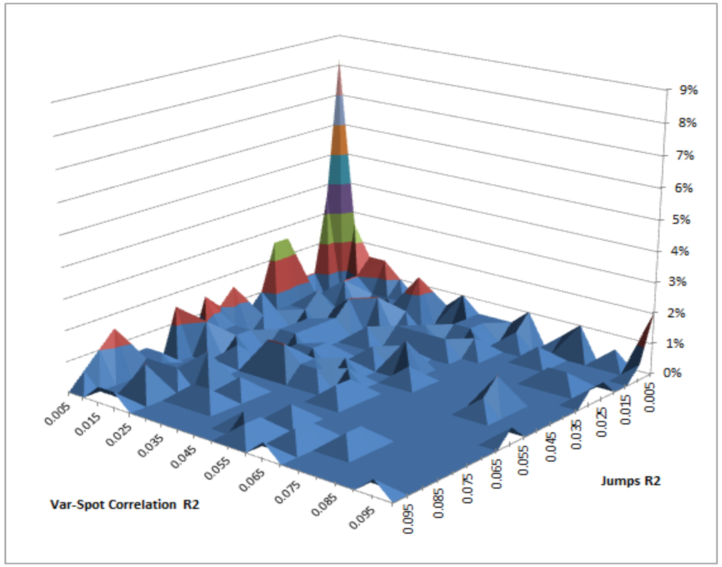
Volatility as a common driver: conclusions

- We found some evidence of dependence between volatility and both CDS spreads and equities
  - Usually strongest for ATM vols, skew effects for levels but not for returns
  - Better for CDS levels than for CDS returns, but conversely for equities
- Some valuable information gathered
  - Dependence between levels can be useful for longer-term links (although not quite working for equity)
  - Confirmation of change in the dependence for extreme returns (although need a better model to capture it properly)
- Overall, the dependence is generally not strong enough to build a model around
  - Weakest for historical vols, which has the most importance for risk models
- Cannot reliably conclude that volatility can be modelled as a common driver behind equity and credit underlyings
- Unlikely that the situation is any better in the tails…
- Volatility as a guide to equity modelling language (jumps vs. correlation) - some indicative findings in favour of jumps – worth a separate look

Historical information on skew: measure by regression

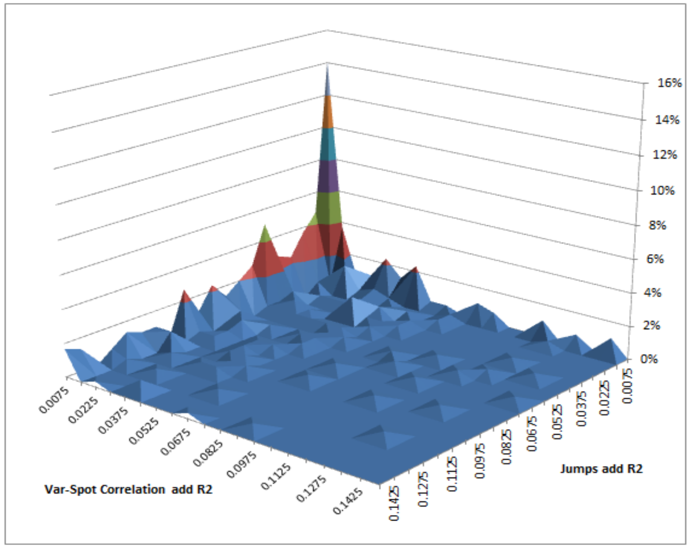
- Recall two historical measures of equity volatility skew information:
  - Correlation between historical equity returns and their variance (inspired by Heston and Heston-Nandi stochastic volatility models)
  - Historical jump statistics, such as time-averaged historical intensity times average size of jumps (inspired by jump-diffusion models)
- Regress implied on historical volatilities, compare R^2's
  - ATM on historical vol levels dependence is high (60% median R^2)
  - Regress OTM skew on various historical “proxies”: weak dependence overall, but jumps give more high values (median R^2 at 9%, vs. 2-3% for correlation proxies)

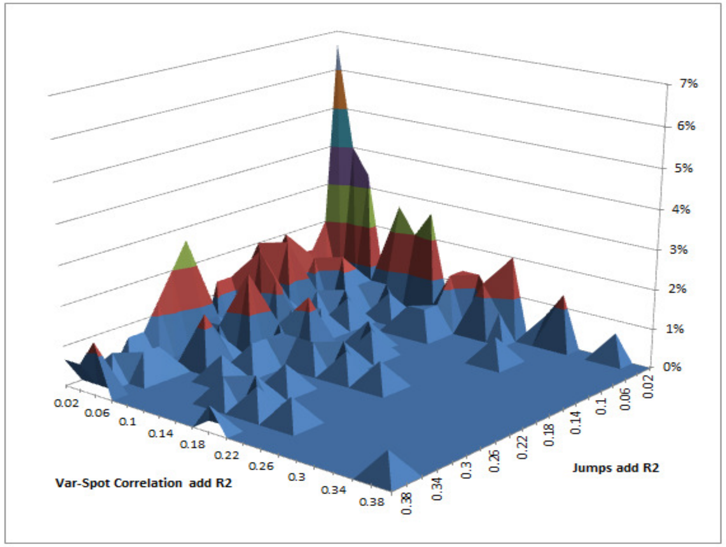




Historical skew information: correlation, jumps or both?

- 3D plots of the increase in R^2's when historical skew "proxies" are added for CDS (left) and equity (right) *levels*
  - To assess whether correlation (~stochastic volatility models) or jumps (~jump-diffusion models) are more useful for capturing dependence on "historical skew"
- For non-zero contributions, clustering is around the axes, rather than in the middle, implying that one effect usually dominates
- More clustering around the "jump" axis, so jumps look more significant
  - Equity level plot (right) in particular suggests that **jump-diffusion language is better than stochastic volatility in historical measure**



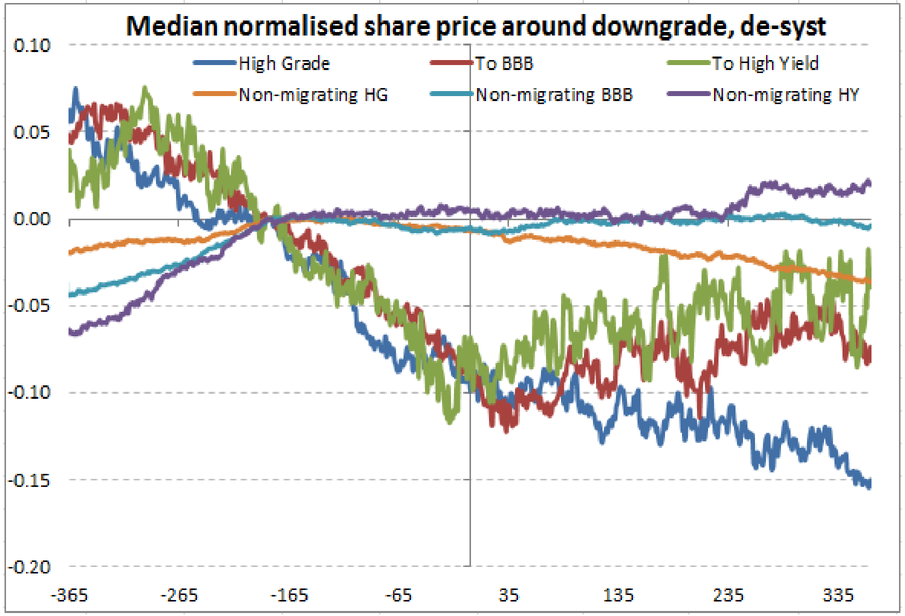


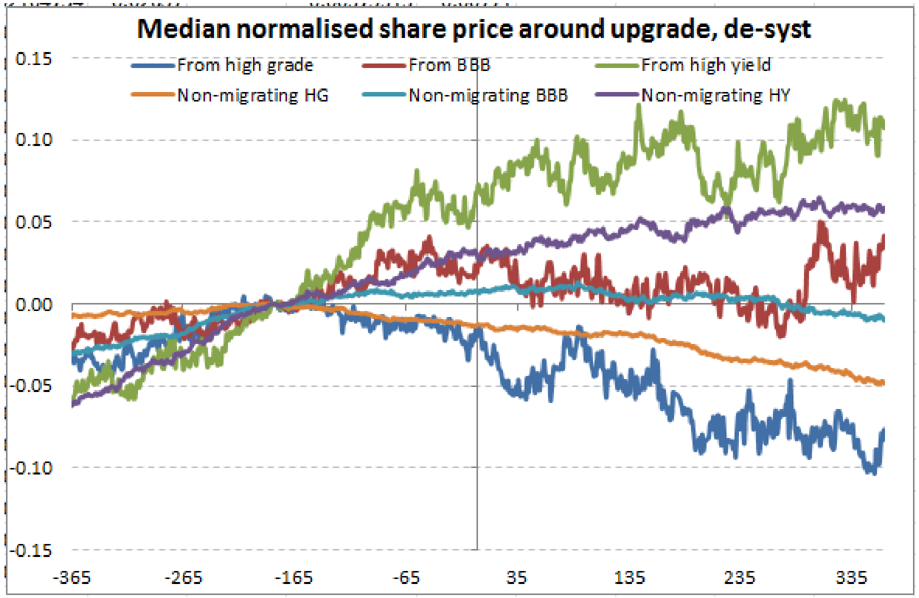
Revisit structural link between equity and credit

- "New flavour" did not work, so back to the standard common driver: Merton
- Structural approach: credit and equity are driven by asset returns (call/put)
- Observable quantity which can be related to asset returns: rating
- Ratings: is it just a convenient discretisation…
  - Moody's KMV and similar: ratings change when asset returns cross thresholds
  - Historical transition probability tables provide a calibration vehicle for discretised asset return models
- … or a fundamental property of asset return evolution?
  - Are asset return dynamics continuous or event driven?
  - Does the market take ratings into account, as reflection of firm-value relevant events, or are they arbitrary discretisations?
- Question: how do share prices and credit spreads react to rating migrations?
  - Agency rating actions are likely to trail the market
  - Need to observe behaviour before and after downgrades and upgrades
- Centring around the migration event, look at averaged share price, CDS spread and implied volatility behaviour – is it a "driving event"?
- Next pages: agency rating changes are clearly reflected in market behaviour

Share price dynamics around rating migrations

- Downgraded names – "hockey stick" pattern: negative drift before, stable after
- Starts approximately 9 months before the event



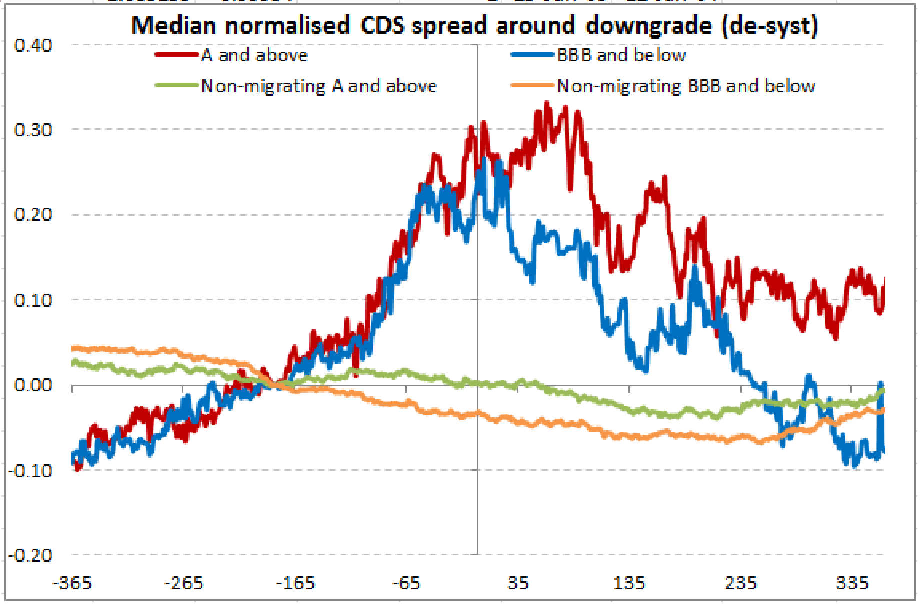


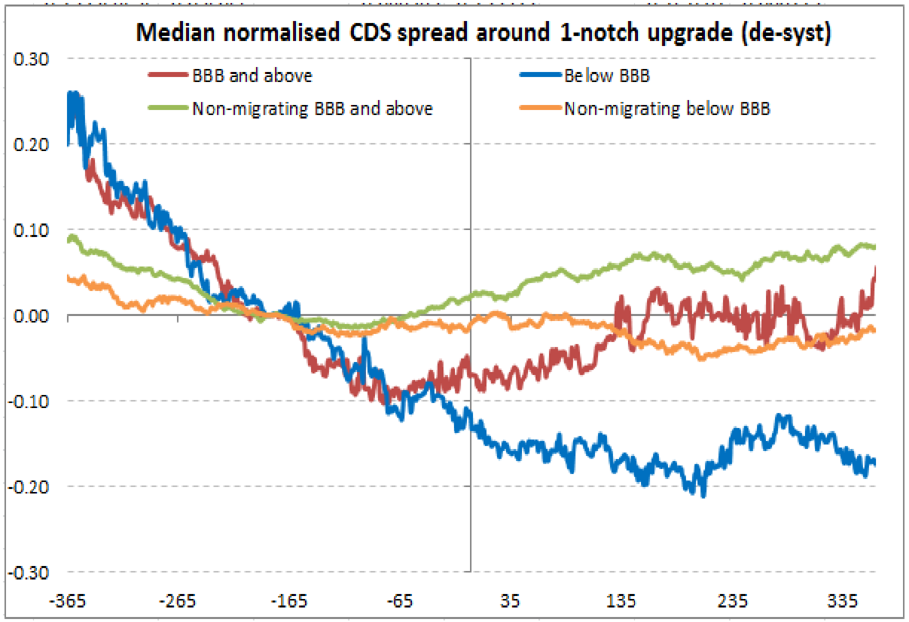
- Upgraded names: smaller upward drift before, largely stable after
- Timing less clear, possibly a slower and/or weaker effect
- "Risk-return" pattern after the event decreases if "de-systematised" per rating band

* "De-syst" means that market average has been subtracted

CDS spread dynamics around rating migrations

- Downgraded names – "hat" pattern: spreads rise before, drop after
- Post-downgrade level higher, reflecting increased credit risk

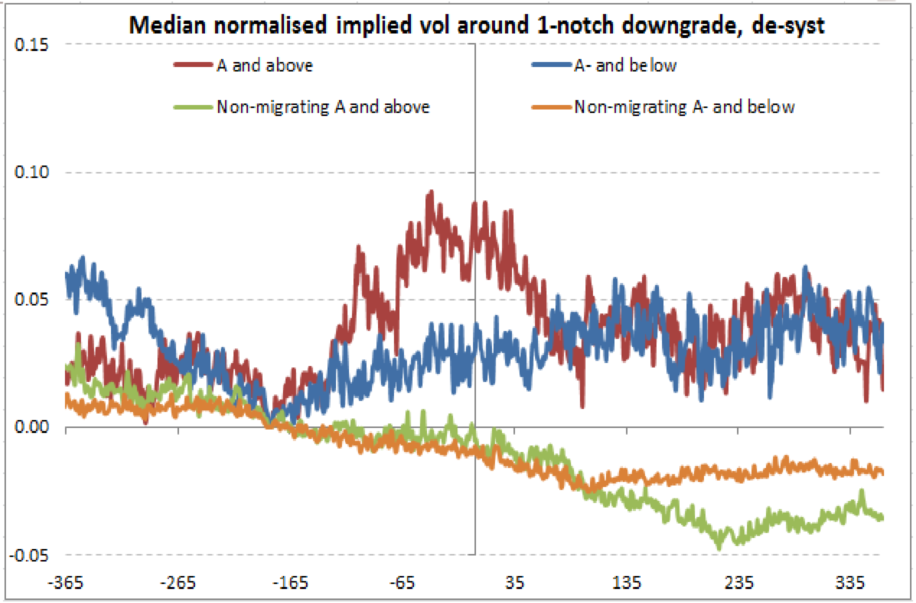


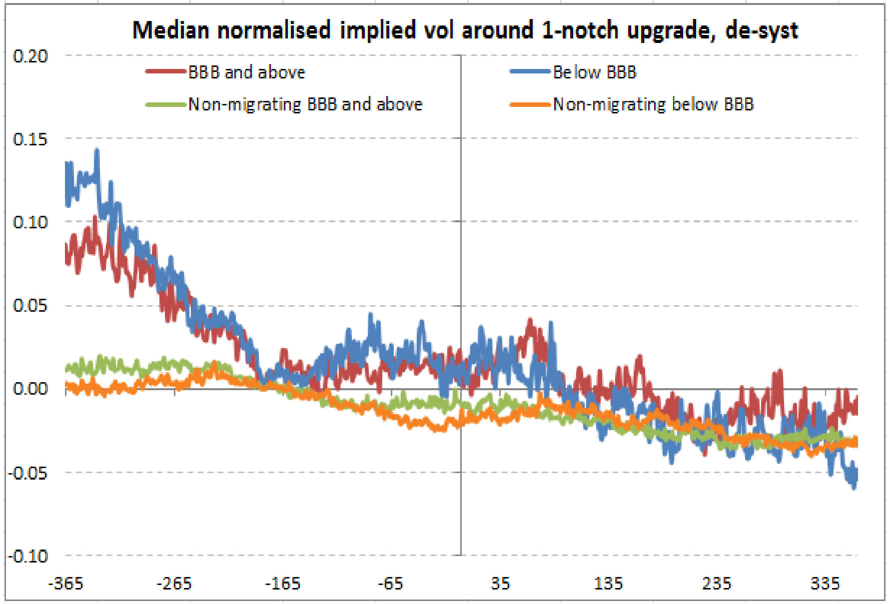


- Upgraded names: less clear, some hybrid of "hockey stick" and "hat" patterns
- Signal weaker overall

Implied volatility dynamics around rating migrations

- Downgraded names: similar to CDS ("hat" pattern), stronger for highly rated names
- Some unexpected pre-event drifts detected as well

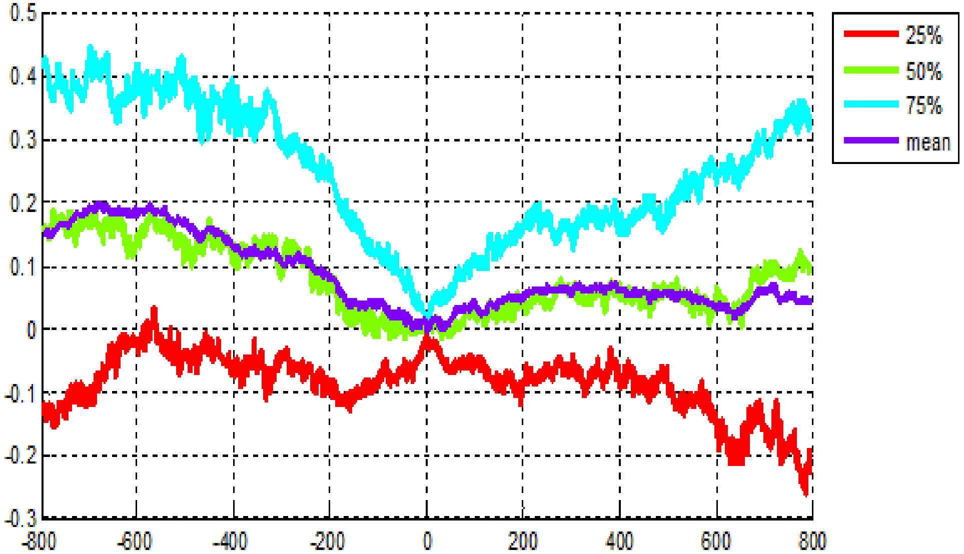


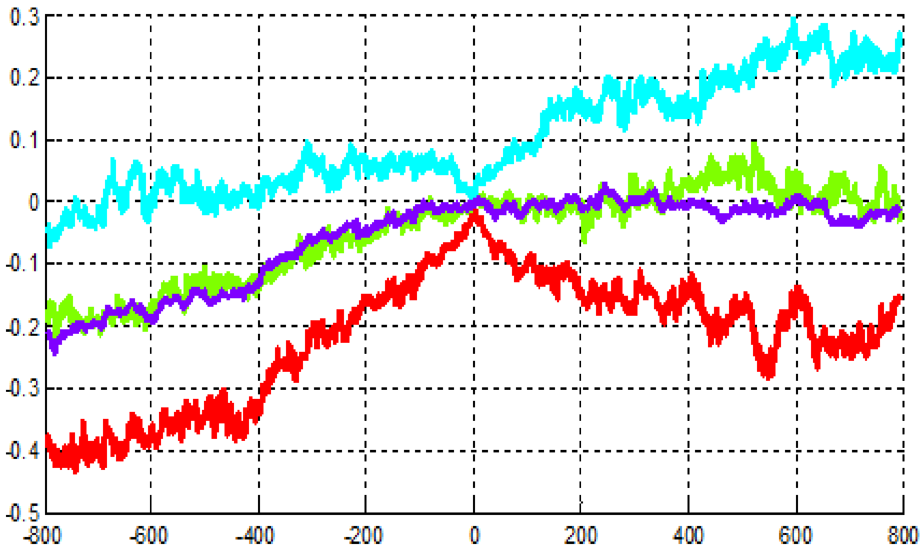


- Upgraded names: "hockey stick" pattern, implied vol dropping and staying low through the event

Asymmetry in the market?

- Share price reactions to downgrades vs. upgrades appears to differ in strength
  - Downgrades preceded by 6-9 month of negative drift, ~20% annualised
  - Positive drift before upgrades less significant, at most 5% p.a. over the same period
- Bad news for Merton's model?
  - Example: consecutive rating changes of up 1 notch, then down 1 notch, vs. down 1 notch, then up 1 notch
  - Should come back to the same price in the model – but not in the market?
- Evidence over longer term (3-5 years) - Merton model takes over





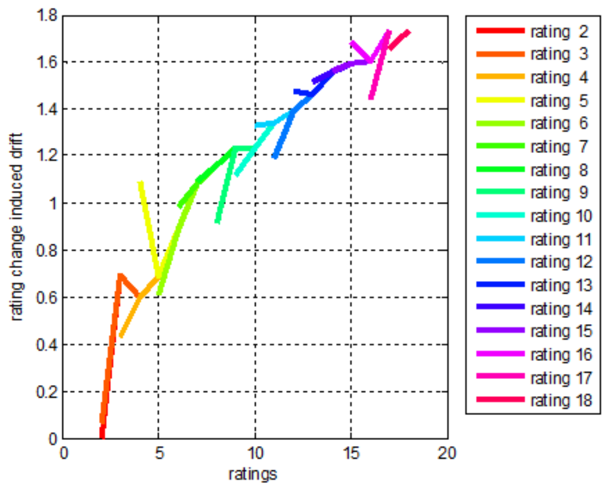
Symmetry: definition and model advantages

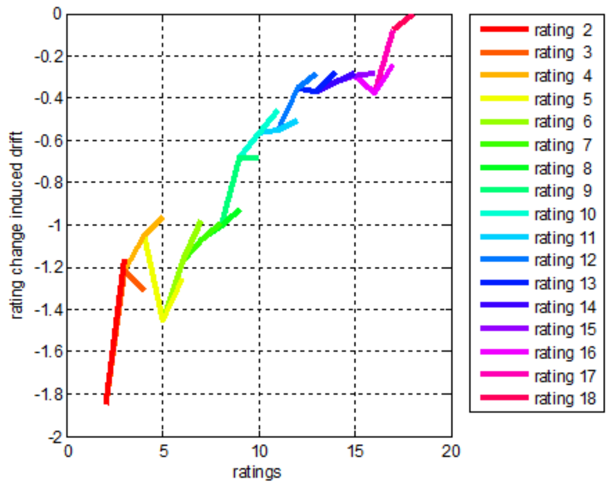
- Merton model takes over at long horizons…
  - Just established: +1 -1 = -1 +1 *on average*
  - What about 0 +2 = +1 +1 etc.? Approximately, yes (within error margin)
- Why are these two "yeses" important?
  - Allows to construct a single "share price-to-rating" map.
    - Recall that "spread-to-rating" maps are well known in IRC/CRM modelling and allow construction of "simple" models on Lego principles
  - Instead of mapping *ratings to levels (*as for spreads), consider mapping from *rating transition to ratio of share prices* (after and before the event).
  - More formally, the questions to ask:
    - Initial rating dependence: moving from rating ***a*** to rating ***a+k*** vs. from rating ***b*** to rating ***b+k***
    - Path dependence: intermediate steps in moving from rating ***a*** to rating ***b***
    - Direction dependence: from ***a*** to ***b*** vs. from ***b*** to ***a***. (Can be subset of path dependence)
- Excluded time dimension:
  - Is speed (drift) different between up- and downgrades only or are there other dependencies, e.g., investment/non-investment grade, high/low volatility bands etc.?
    - Plots on previous pages show that there is some dependence
    - Assuming it does exist will not affect portfolio risking in a material way
- Graphical visualisation: introducing "tree branches", "needles and "twigs"…

"Tree branches", "needles" and "twigs"… (I)

- Introducing the toolkit – look at single transition only at first:
  - Cumulative equity drift of migrated names vs. drifts of those names that did not migrate
    - In each rating category, take average growth rate of the names that stayed in this rating over the whole 5 year period as "base"; calculate average excess drift of equity log-returns for the names that migrated out of this rating
    - Assume that consecutive migrations lead to adding up [log] drifts, plot the cumulative drift as a function of rating, increasing for consecutive upgrades or decreasing for downgrades: "*spine curve*"
    - Consider "spine curve" as the "path" of cumulative equity drift from consecutive upgrades. For each rating category on the upgrade drift "path", estimate excess drift averaged over observed single-notch downgrades from this rating and draw a line on top of "spine curve" (left-hand panel) – this is the extra "needle" or "twig".
    - **If upgrade and downgrade effects were completely symmetric, no "needles" or "twigs" would exist**

Rating n	*S&P*
2	[C-CCC]
3	B-
4	B
……………………	
17	AA+
18	AAA

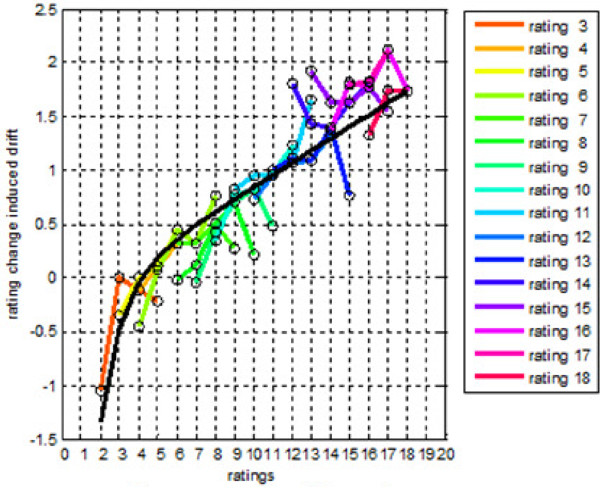




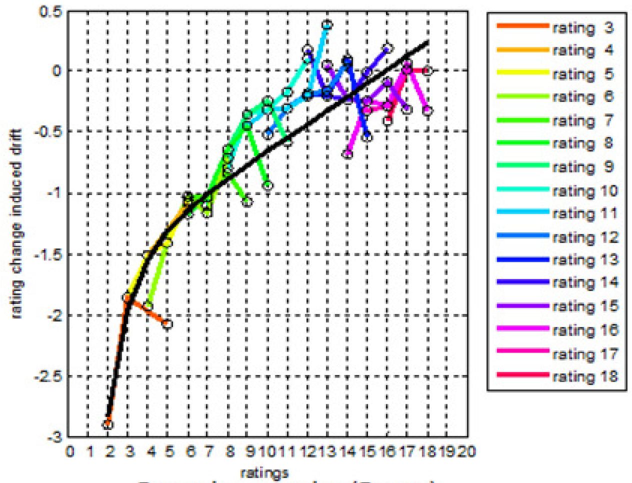
"Tree branches" , "needles" and "twigs"… (II)

- Add "twigs" corresponding to excess drift estimates from double-notch upgrades and downgrades
- Add 3-year version
  - Some cleaning up
    - Remove outliers for small numbers of observations with unexpected results (e.g. large negative drift estimates from 3 to 4 and 4 to 5 upgrades)
  - Use "Mk1 Eyeball" or fit one pane and impose on others:
    - Fit 1-notch downgrades over 5 years with $\mu(R) = a_1 + a_2 R + a_3 e^{-a_4 R + a_5}, \quad \mu = \log(S/S_0)$
- Now we jump to the next slide to see the "forest"….

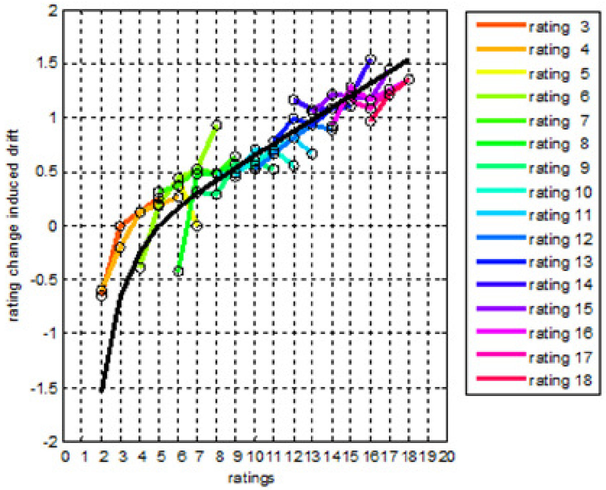
“Tree branches” , “needles” and “twigs”… (III)



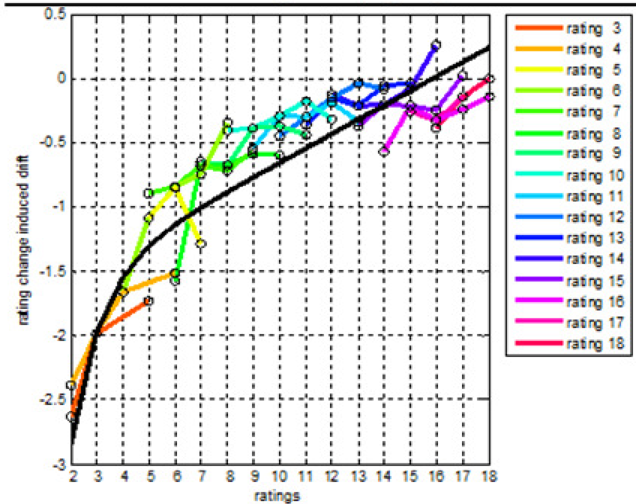
From upgrades (5 year)



From downgrades (5 year)



From upgrades (3 year)



From downgrades (3 year)

“Tree branches” , “needles” and “twigs”… (IV)

- Add twigs corresponding to excess drift estimates from double-notch upgrades and downgrades
- Add 3-year version
  - Some cleaning up
    - Remove outliers for small numbers of observations with unexpected results (e.g. large negative drift estimates from 3 to 4 and 4 to 5 upgrades)
  - Use “Mk1 Eyeball” or fit one pane and impose on others:
    - Fit 1-notch downgrades over 5 years with $\mu(R) = a_1 + a_2 R + a_3 e^{-a_4 R + a_5}, \quad \mu = \log(S/S_0)$

And we are back to the discussion…

- “Needles” or “twigs” are “sticking out”, but
  - There is no evidence of systematic difference, either: “needles” stick out in random directions, on both sides of the “spine curve”
  - Conclusion: even though for any given rating, excess drift estimates from upgrades and downgrades are different, the best match for all ratings at the same time is likely to be the assumption of equal drift from upgrades and downgrades.
    - Then price is given by $S^{+} = S^{-} \exp\big(\mu(R^{+}) - \mu(R^{-})\big)$, where $\mu(R)$ represents the fitted share price cumulative drift, for a given rating, for a migration from rating *R-* to rating *R+*

The "migration" Lego

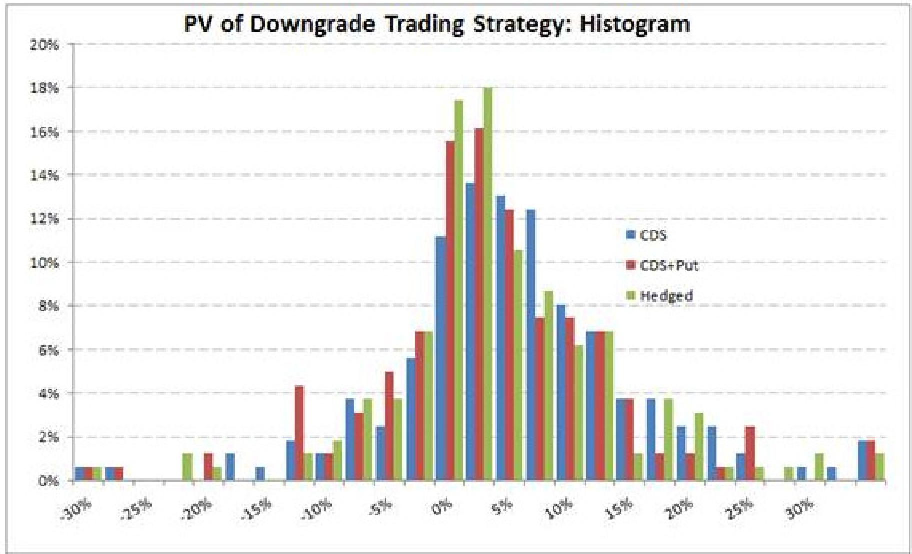
- Advantages of having a single-parameter family, or a single share price-to-rating *curve*, rather than a complicated general mapping are numerous:
  - Long-term symmetry between upgrade and downgrade effects on share price is preserved by construction
  - Simulated equity prices at maturity do not depend on rating paths, so no artificial path dependency is introduced into vanilla equity derivatives products
  - Sizes of drift corrections and/or jumps caused by different kinds of rating migrations are consistent
  - Estimates of the mapping itself will be more reliable, as rating categories which produce more observations naturally contribute more to the fitting
- Any missing bits?
  - Neither of the Lego bricks (diffusion earlier, migration here, jumps later) includes a discussion of wrong/right way risk, in another words, conditionality on default
  - The general philosophy is applicable, but this part of Lego set lies outside of this presentation (watch this space…)
  - A few teasers
    - Equity instruments are more often path dependent than credit derivatives, so the approximation should handle conditional paths
    - A defaulted name can "conditionally" come back to life – Zombies!

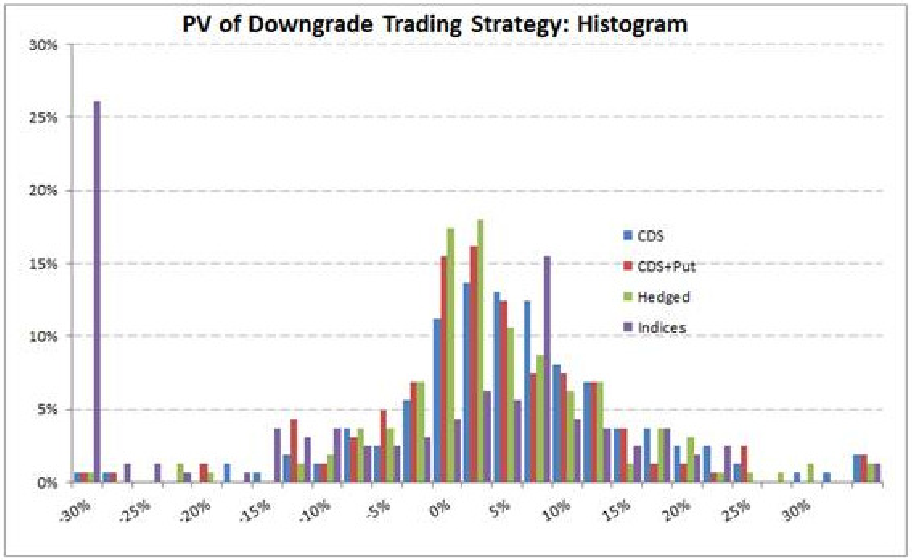
Rating transitions, CDS and equity: can we make money?

- Design trading strategy to exploit CDS and equity behaviour around migration
  - Obviously, non-anticipating (e.g., cannot short stock ahead of downgrade)
- Key points around rating downgrade:
  - CDS spread widens up to migration date, then tightens $\Rightarrow$ sell protection
  - Equity price falls up to migration date, then is stable $\Rightarrow$ use put as “crash” hedge
  - Cleaner on idiosyncratic equity returns $\Rightarrow$ can add opposite index positions
  - Risk horizon is 6 months to 1 year, but need to be mindful of liquidity
- Strategies to explore: when an issuer downgrade is announced…
  - Sell 5y CDS protection on it, unwind in 6 months (*naked CDS*)
  - As above, + buy 6m or 1y 50% OTM equity put, unwind in 6 months (*CDS + put*)
  - As above, + statically hedge put by selling the underlying; + sell 1y ATM equity index put to finance single-name equity put (i.e., premium neutral), statically hedged by buying index; unwind all in 6 months (*name + index, or hedged strategy*)
- Different risk profiles for these strategies:
  - Naked CDS is directional, exposed to default risk
  - OTM put: add protection against default, which is cheap if our pattern is realised
  - Offsetting index positions finance single-name equity put position and take out market-wide trend effects

Trading strategies: testing performance

- Compare to a suitable benchmark
  - Theoretical: mean/median excess return over the risk-free rate
  - Practical alternative: invest €1 equivalent, in equal measures, into five major equity indices (S&P 500, FTSE 100, Nikkei 225, CAC 40 and DAX 30), for 6 months
- Tested on 10 years' worth of data: 2003 – 2013
  - Total of 161 single-notch downgrades, on 130 names
  - Improved strategy: use names whose CDS spread at the time of downgrade was higher than average for the rating





Trading strategies: performance

- On average, “downgrade” trading strategy is profitable
  - Roughly twice as many money-making names as money-losing ones (ratio is larger for naked CDS, smaller for CDS + put)
  - Average PV is 5% upfront (median is 3.25%) for naked CDS, 3.25% (1.5%) for CDS + put and 3.8% (1.8%) for index + name hedged strategy
  - Standard deviation of 14.5% for naked CDS, reduced below 14% for the other two
- Average Sharpe ratios:
  - 0.28 for naked CDS
  - 0.16 for CDS + put
  - 0.2 for name + index, or hedged strategy
  - Index positions take out systematic effect, making “signal” stronger
- If CDS with lower-than-average spread for rating are used, strategy deteriorates
  - naked CDS only gives 0.1% PV on average, hedged strategies lose money
- Survival bias: defaulted names are not included
  - Hurts naked CDS strategy
  - Should be neutralised by the presence of OTM put

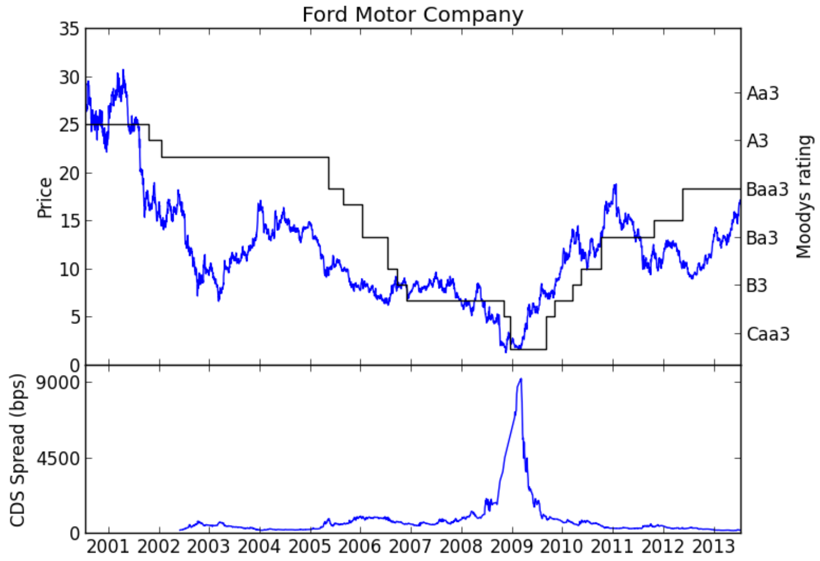
Trading strategies: compare to global equity indices

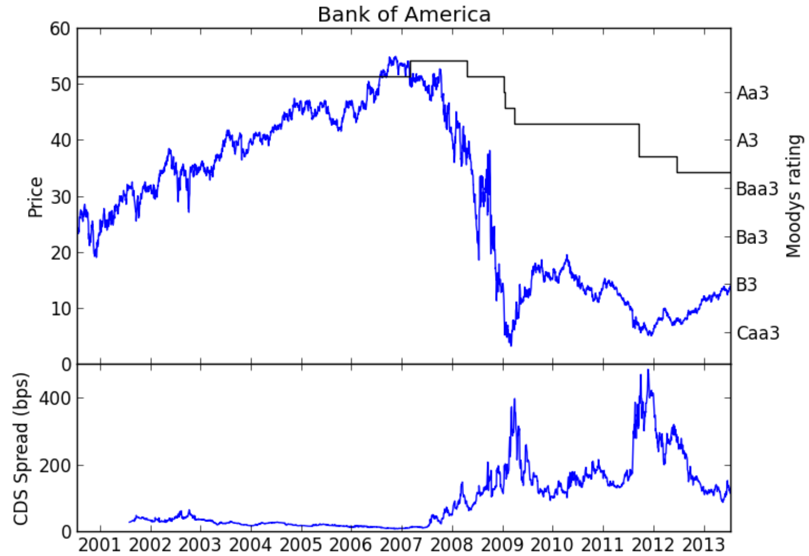
- “Global equity index” strategy performs really badly
  - Sharpe ratio is -0.43 with respect to risk-free rate
    - Compared to +0.2 for the name + index “downgrade” strategy
  - Even though it makes money about half the time, average PV is almost -8%!
  - Median is -1.5%: performance is dominated by a number of catastrophic losses (also see distribution)
  - Understandable given the period chosen (2003 – 2013), but poor as benchmark

- This illustration supports the view that a good risk model and a good arbitrage model should have a common root

Rating migrations and jumps

- We have seen that on average, both CDS spreads and share prices react smoothly to rating migrations, so we introduce a “Lego brick for drift”
- However for individual names, rating migrations can induce jumps
  - Limit case: share price goes to zero on default
  - Evidence of excess kurtosis in spread distributions following migrations

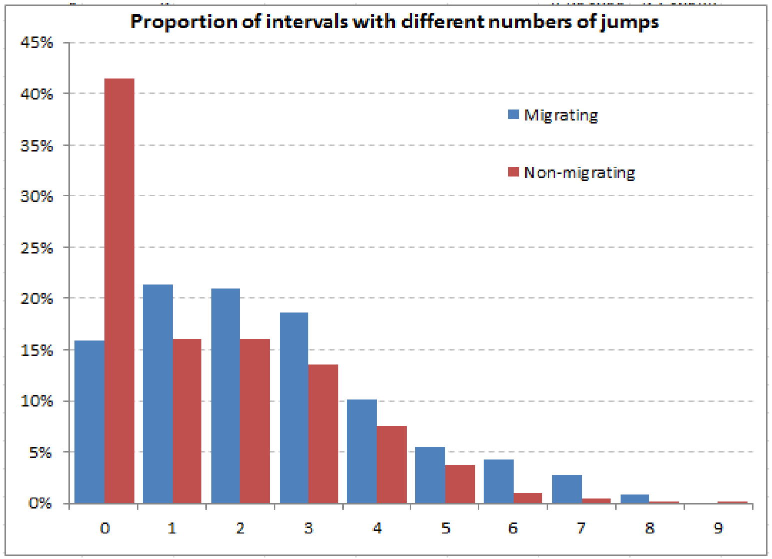


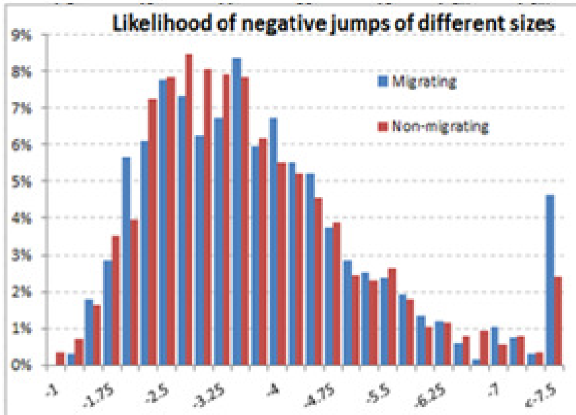


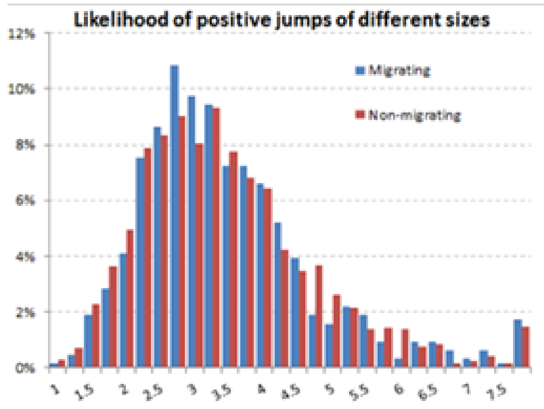
- CDS spreads: long-term mean levels differ by rating
  - Upon migration, change mean reversion level and/or add jump to fatten tails
- What about equities?

Jumps in equity returns and effect of migrations

- Analyse a large family of equity price time series, by 1-year periods
  - 335 names, daily returns over 2002 – 2013, ~900 total rating migrations
  - Detect jumps in equity returns using Lee-Mykland algorithm, ~8000 jumps found
  - Clearly, 9 times more jumps than migrations, but do rating transitions affect the likelihood, size and/or intensity of jumps?







- Clearly, it is more likely for equity return to
  - jump if migration occurred
  - have large negative jumps if migration occurred

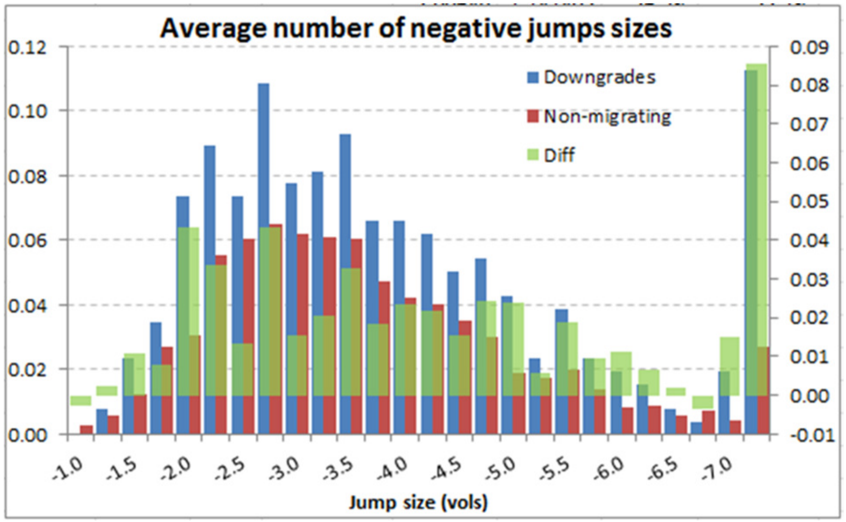
Jumps in Lego world (I)

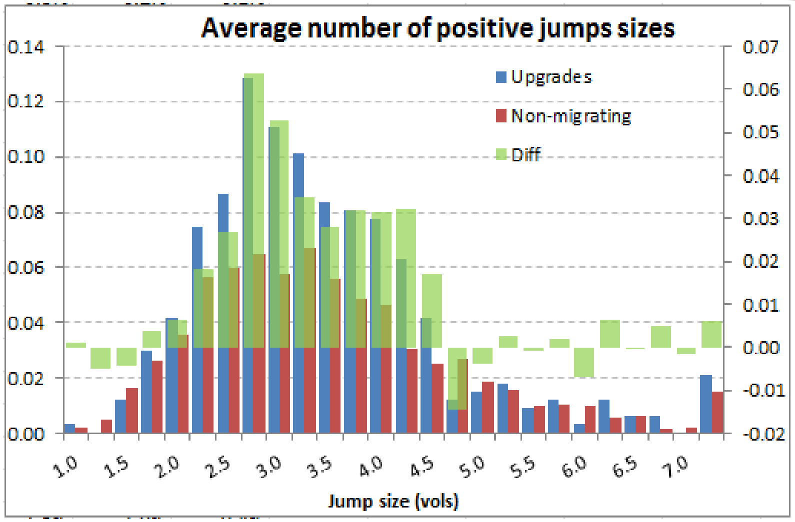
- Returning to "assembly" approach, what alternative Lego bricks exist?
  - Need to ask different type of questions for different brick types
  - Full jump model: rating change affects jump process parameters
  - Can extreme moves be introduced without investing in jump processes?
    - Most extreme moves are unlikely to be disconnected from rating change
    - Lego by "risk cause"
- Look at jumps without normalisation (absolute frequency)

	Non-migrating	Upgrades	Downgrades
Number of intervals			
- total	2338	335	258
- without jumps	969	53	41
- with jumps up	1002	206	159
- with jumps down	1057	208	167
- *with both up and down jumps*	*690*	*241*	*109*
Number of jumps			
- total	3472	694	612
- up	1675	351	285
- down	1797	343	327
Jumps per interval			
- total	1.485	2.072	2.372
- up	0.716	1.048	1.105
- down	0.769	1.024	1.267

Statistics of equity jumps over intervals with and without migration

Jumps in Lego World (II)





- This justifies inclusion of extreme migration-induced equity jumps
  - Example: introduce a downward jump with downgrade by three or more notches. From Moody's 1y transition probability matrix,
    - 3- -notch downgrade without defaulting is at most 4.5% for any starting rating, which is much less than any size of right tail of the distribution (blue bars)
    - 3+-notch upgrades and upward jumps: 2.1% per annum - same conclusion
    - We can associate a jump of almost any size with events that happen as rare as 3-notch downgrades without adding "too much" tail risk
  - Another Lego brick: 4.5% is smaller than even the tail of the *difference*, so the corresponding jump can be a *downgrade add-on* for existing equity jumps process

Conclusions (I)

- Credit-equity models can be built in blocks

- Blocks can be built separately and implemented separately without significant loss of quality of the overall model

- Blocks can be built around risks, not assets. Examples:
  - Suitable jump-diffusion components for each underlying
  - Correlated diffusions to capture short-term [negative] correlations between returns
  - Common jumps for more accurate description of joint tail events (high percentiles)
  - Long-term links: drift modifiers, mean-reverting spreads, common drivers

Conclusions (II)

- Fat-tailed equity returns are better explained by jumps than by stochastic volatility
  - Integrated of jump size and intensity contains more historical information about volatility skew than correlation between equity returns and their variance

- Volatility as a common driver behind equity and credit
  - May link levels or returns, depending on interpretation (leverage or default probability)
  - Empirical links not sufficiently strong, especially when calibrated to historical data

- Rating migrations reflect market events or firm's "regime shift", they are not just convenient discretisations of structural models (although discretisation is still useful!)
  - Spreads, share prices and volatilities all react to rating migrations
  - Predicted dynamics of these market factors can be monetised
  - Evidence of change in drift and jump behaviour around rating transitions on single-name level

Acknowledgements

- Thanks to Erdem Ultanir, Vera Minina, Mirela Predescu, Jean-Baptiste Brunac, Sascha Wilkens, Cecile Barthelemy and Rim Tehraoui for their input at various stages of this work.

Disclaimer

The views expressed by authors in this presentation are their own and do not necessarily reflect the views of BNP Paribas.

This presentation is for information and illustration purposes only. It does not, nor is it intended to, constitute an offer to acquire, or solicit an offer to acquire any securities or other financial instruments.

This document does not constitute a prospectus and is not intended to provide the sole basis for any evaluation of any transaction, securities or other financial instruments mentioned herein. To the extent that any transactions is subsequently entered between the recipient and BNP Paribas, such transaction will be entered into upon such terms as may be agreed by the parties in the relevant documentation. Although the information in this document has been obtained from sources which BNP Paribas believes to be reliable, BNP Paribas does not represent or warrant its accuracy and such information may be incomplete or condensed.

Any person who receives this document agrees that the merits or suitability of any transaction, security or other financial instrument to such person's particular situation will have to be independently determined by such person, including consideration of the legal, tax, accounting, regulatory, financial and other related aspects thereof. In particular, BNP Paribas owes no duty to any person who receives this document (except as required by law or regulation) to exercise any judgement on such person's behalf as to the merits or suitability of any such transaction, security or other financial instruments. All estimates and opinions included in this document may be subject to change without notice. BNP Paribas will not be responsible for the consequences of reliance upon any opinion or statement contained herein or for any omission.

This information is not tailored for any particular investor and does not constitute individual investment advice. This document is confidential and is being submitted to selected recipients only. It may not be reproduced (in whole or in part) or delivered to any other person without the prior written permission of BNP Paribas.